AERODYNAMICS OF WING WITH SUBMERGED LIFTING FAN

BY

VIJAY SARIHAN

DEPARTMENT OF AERONAUTICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
FEBRUARY, 1973

AERODYNAMICS OF WING WITH SUBMERGED LIFTING FAN

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

BY

VIJAY SARIHAN

to the

DEPARTMENT OF AERONAUTICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
FEBRUARY, 1973

CERTIFICATE

Certified that this work on 'AERODYNAMICS OF WING WITH SUBMERGED LIFTING FAN', has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

N L Arora
(N.L. Arora)
Dept. of Aeronautical Engineering
Indian Institute of Technology,
Kanpur

ACKNOWLEDGEMENTS

The topic of this thesis was suggested by Dr. S.M. Ramachandra. I am indebted to him for his guidance, help and encouragement during the initial stages of this thesis. I am indebted to Dr.N.L.Arora for his guidance and help in the later stages of this thesis.

I am thankful to Mr. C.V.R. Murti and Dr.M.Krishnamoorthy for their valuable suggestions and discussions from time to time. Thanks are due to all my friends, especially Mr. B.S. Bhadoria, for their immense help.

Thanks are also due to the staff of the Computer Centre without whose cooperation this work would have been incomplete.

I am grateful to Mr. S.R. Gupta for his excellent and patient typing.

I am grateful to the Ministry of Defence, Government of India who supported this project under the Grant-in-Aid schemes on 'Wing Body Interference and Stability Derivatives'.

VIJAY SARIHAN

CONTENTS

ABSTRACT

A theoretical model for analysing the wing with submerged lifting fan is developed to determine the theoretical aerodynamic characteristics of vertical/short take-off and landing (V/STOL) type aircraft. The model replaces the wing by a flat plate which is replaced by a net work of horse-shoe vortices distributed on its surface and trailing behind. For the ducted fan a series of singularities representing the fan disc, duct, centre-body and the wake are used. Flow tangency condition on the wing is used to determine the wing singularities and an iterative procedure is used for the ducted fan analysis. Both the effect of the wing on the fan and the fan on the wing are considered. The effect of the fan on the wing is obtained in a simplified manner by considering only the duct bound vorticity. The effects of all other ducted fan singularities are assumed to be small in comparison. The theoretical results obtained are compared with experiment.

A computer program in Fortran IV for use on IBM 7044 is developed for the computer simulation of the problem. It uses an existing NASA program for the analysis of the ducted fan in cross-flow.

SYMBOLS

For the Wing and the Combined Fan-Wing Model:

A	aspect ratio, $4b^2/S$
b	semi-span of the wing
c	local chord
$\bar{c}_A$	aerodynamic mean chord, $\frac{2}{S}\int_0^b c^2\,dy$
C_F	propeller force coefficient, $T/q_\infty S$
C_l'	local lift coefficient, Eq. (2.9)
C_l	conventional local lift coefficient, $\frac{\text{Lift(local)}}{q_\infty c}$
C_L'	lift coefficient, Eq. (2.15)
C_L	conventional lift coefficient, $\frac{\text{Lift}}{q_\infty S}$
C_m'	local moment coefficient
C_m	conventional local moment coefficient, $\frac{M(\text{local})}{q_\infty c^2}$
C_M'	moment coefficient, Eq. (2.16)
C_M	conventional moment coefficient, $\frac{M}{q_\infty S\bar{c}_A}$
C_{p_l}	lower surface pressure coefficient, $\frac{p_l-p_o}{q_\infty}$
C_{p_u}	upper surface pressure coefficient, $\frac{p_u-p_o}{q_\infty}$
C_p	coefficient of the pressure difference across the wing surface, $C_{p_l}-C_{p_u}$
C_R	wing root chord
$E(k)$	complete elliptic integral of the second kind, $\int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\;d\theta$
f	nondimensional down-wash function, Eq. (2.6).

F	nondimensional down-wash function, Eq. (2.7)
$F(k)$	complete elliptic integral of the first kind, $\int_0^{\pi/2} \frac{1}{\sqrt{1-k^2 \sin^2 \theta}} d\theta$
L	lift
M	moment
n	matrix order or number of vortices
p	static pressure
q_∞	free stream dynamic pressure
$\underline{R}$	radius vector
s	vortex semi-span, non-dimensional with respect to C_R
T	fan thrust
v	induced velocity
V	Forward velocity
w	downwash velocity
x	chordwise coordinate, non-dimensional with respect to C_R or axial distance measured from duct c/2
x'	axial distance measured from duct c/2
Δx	fraction of chord over which C_p is assumed constant, non-dimensional with respect to C_R
X	chordwise location of the control point with respect to vortex centre, x_p-x_v
y	spanwise coordinate, non dimensional w.r.t. C_R
Y	spanwise location of the control point with respect to vortex centre, y_p-y_v
Y'	spanwise location of the control point with respect to the image vortex centre, y_p+y_v

α_w	angle of incidence of the wing, radians
α_f	angle of incidence of the fan with respect to the fan axis of rotation, radians
Γ	circulation
ρ	air density

Matrices:

$[\ ]$	a square or rectangular matrix
$\lceil\ \rfloor$	a diagonal matrix
$\{\ \}$	a column matrix
$\{C_p\}$	matrix of pressure coefficients
$[F]$	matrix of downwash influence coefficients
$\{w\}$	matrix of downwash velocity at the control points
$\lceil x' \rfloor$	matrix of distances (in root chords) of the vortex centre from the line about which the moment, are to be determined.
$\|\ \|$	modulus

Subscripts:

f	fan
i	matrix element identification by row
j	matrix element identification by column or jet
l	lower surface or local
le	leading edge
o	ambient
p	control point or propeller
r	radial

te	trailing edge
v	vortex centre
w	wing
$\bar{\ }$	vector

For the fan:

A	area of duct exit plan, $\pi D^2/4$
A_p	propeller disc area, $\pi(R_p^2-R_{CB}^2)$
b	propeller chord length, Fig.9.
c	chord length of the duct,
C_n	Glauert series coefficients for γ_D, Eq. (3.6)
C_p	pressure coefficient
C_l	lift coefficient for fan blade section
C_{l_α}	lift curve slope for fan blade section
D	diameter of duct in the exit plane
D_p	propeller diameter
h	fan blade thickness
i	incidence angle of fan blade section measured from the line of zero lift, Fig. 9.
J	advance ratio, $\frac{V}{n\ D_p}$
l_{CB}	length of centre body
n	fan rotational speed, rev./sec.
q	free stream dynamic pressure
R	radius of duct exit plane, D/2
R_p	radius of fan tip
$\bar{r}$	mean radius of equal area element of fan disc.
S	wing area

t	duct thickness
u	induced axial velocity
u_{CB}	axial velocity induced by centre body singularity distribution
u_{q_D}	axial velocity induced by duct thickness distribution
u_{γ}	axial velocity induced by the vortex cylinder trailing from the duct trailing edge
u_{γ_D}	axial velocity induced by duct-bound vorticity, γ_D
u_{γ_W}	total axial velocity induced by all internal vortex cylinders trailing from the fan
V	free stream velocity
v	induced radial velocity
v_{CB}	radial velocity induced by centre body singularity distribution
v_{γ}	radial velocity induced by the vortex cylinder trailing from the duct trailing edge
v_{γ_W}	total radial velocity induced by all the internal vortex cylinders trailing from the fan
v_{γ_D}	radial velocity induced by duct-bound vorticity, γ_D
x	axial distance from duct leading edge
z	number of equal area annuli making up fan disc area
α_f	free stream angle of attack with respect to the fan axis of rotation,
β	fan blade section pitch angle, Fig. 9.

Γ	circulation bound to fan blade, Eq.(3.1)
ε	convergence criterion
γ	strength of outer trailing vortex cylinder
γ_D	axially symmetric component of duct bound vorticity, Eq. (3.6)
γ_w	strength of w-th inner trailing vortex cylinder
γ_α	duct bound vorticity component due to angle of attack, Eq. (3.8)
θ	transformed axial distance, $x = \frac{1}{2}(1-c \cos \theta)$
ρ	free stream density
ϕ	azimuthal angle
ω	fan rotational speed, rad/sec.

CHAPTER I

INTRODUCTION

Of all the known aircraft configurations the fixed wing aircraft is the best logistically, tactically and economically. Its utility would be greatly increased if it could also take off and land vertically, hover, and move to the side and rear at low speeds. This can be achieved if the forward flight aerodynamics of the aircraft is not compromised and an efficient method of converting the propulsive power to vertical lift is designed. Amongst the many VTOL configurations, the fan in wing system is very promising because it enhances the utility of the aircraft with the least compromise to its present characteristics. This system consists of a ducted fan or fans submerged in the wing or the fuselage with an arrangement for the distribution of the propulsive power to drive the ducted fans. The ducts have an arrangement of vanes which smoothly close the duct inlet and exit during forward flight.

The flight mode of a VTOL aircraft can be divided into three regimes. They are lift-off, transition and cruise. Transition comprises the complete flight regime lying between the static hovering aircraft at one end and the cruising aircraft at the other. The performance of the fan-wing system in forward flight is not very different from the conventional

aircraft. However, the complexity comes in evaluating the aerodynamics during lift-off and transition. This report concerns itself only with the transition regime. From the aerodynamics point of view some of the major problems faced by the fan-wing system during transition are:

1) Ducted fan in cross-flow,
2) Interference effects of fan on wing and wing on fan,
3) Effect of vanes and inlet and outlet shapes of the duct on lifting efficiency of the ducted fan.
4) Interference of free stream with fan jet and its influence on the wing undersurface,
5) The effect of ground proximity on the system.

A considerable amount of experimental work has been carried out on wing/body submerged lift-fans for V/STOL applications. Some of the conclusions, which are relevent to the present study, are presented. Wardlaw and McEachern [3] and Duvivier and McCallum [4] observed that under static conditions the combined lift of the fan-wing system is lower than the thrust of the fan alone. Hickey and Ellis [5] report considerable lift for the wing fan system over the fan thrust. Hackett [6] observed a slight improvement in the static lift due to the attachment of the wing to a fan-nacelle system. References [7-12] do not report any loss of lift for the wing fan combination under static conditions. However, References [5,7,10,11] report a lift loss for the wing-fan system at low flight speeds for a given fan rpm.

Spreeman [10] reports no change in fan-wing total lift with forward speed. Trebble [13] mentions that Wyatt [14,15] observed loss of lift at all forward speeds investigated. Goldsmith and Hickey [16] observed no loss of lift on full scale models. All other experimental work surveyed on the fan-wing system indicates that there is a rapid recovery of the total lift and a subsequent increase even beyond the static lift at zero wing incidence.

The behaviour of the fan-wing system at incidence, the effect of inlet and exit vanes on the total lift, drag, pitching moment and fan power, effect of ground proximity have all been studied in some of the references given above. However, all experimental work suffers from a degree of unreliability because of complicated wall interference effects on models with a near vertical jet efflux. Almost all the results quoted above are without any correction for the tunnel wall interference.

Theoretical studies suggest, in general, that the loss of lift at low forward speeds is due to the fan jet induced suction on the underside of the wing. The subsequent recovery and increase of lift at higher speeds is due to the fan induced upwash in the plane of the wing and the associated additional circulation of the wing. Norland [19] discusses the case of a two dimensional wing with a submerged lifting fan. He replaces the portion of the wing ahead of the hole by an ellipse and the portion behind by a flat plate. The gap between the ellipse and the flat plate is equal to the fan diameter. He

suggests that the lift loss at low flight speeds is due to flow seperation on the under side of the wing behind the fan and the subsequent recovery and increase in lift is due to the reattachment of the flow. He determines the circulation of the ellipse and the flat plate by using the condition of the jet efflux velocity over the fan disc and the Kutta condition at the leading edge or the trailing edge depending on the jet efflux velocity. The main purpose of the paper was to investigate the lift discrepancy-decrease of lift at low V/V_j and a subsequent increase.

Martin [20] extends the two dimensional approach of Norland to a three dimensional case using the lifting line theory. He does not consider the effect of the wing on the fan. The wing is approximated by a discrete number of rectangular sections each having a constant strength bound vortex and a corresponding trailing vortex system. The effect of the fan is considered as an additional induced circulation in the rectangular section containing the fan. The strength of the vortex system is determined by applying the condition of flow tangency on the wing along with the proper jet efflux velocity on the fan disc.

McEachern and Currie [21] replace the two dimensional wing with a submerged lifting fan by a vortex lattice. The strength of the vortices in the lattice is determined by applying the proper surface boundary conditions at control points chosen on the wing surface and the fan disc.

Monical [22] uses the same vortex lattice model for a finite wing but considers the effect of the jet efflux in a different manner. He replaces the jet plume by a curved tube of square cross-section made up of a network of vortices, their strengths being determined by taking proper boundary conditions at a number of control points on the surface of the tube. Both McEachern [21] and Monical [22] replace the wing-fan system by a vortex network. This network though adequate for representing the wing does not represent the fan and the fan jet adequately.

Murti [24] uses a modified lifting line for the wing and tries two different models, a vortex ring and a vortex tube, for the fan. He considers the effect of the fan on the wing which results in a modified lifting line, but does not consider the effect of the wing on the fan. The vortex ring model predicted some fan induced wing lift but the trend was very different from what was observed experimentally [7]. The straight, semi-infinite vortex tube model did not predict any induced lift on the wing. The vortex tube model for the deflected jet gave some induced lift but it was much smaller than what was observed. The results of the above studies agree at best qualitatively with the experimental results.

In this report, a detailed model for the fan-wing system has been proposed and studied. The wing with a submerged lifting fan has been seperated into a wing with a duct and a ducted fan. The wing with a duct has been replaced by

a network of vortices and a series of singularities have been used for representing the fan. A computer program in Fortran IV for use on IBM 7044 has been developed for the computer simulation of the problem. The program uses an existing NASA computer program [18] for analysing the ducted fan in cross-flow.

The total lift produced by a fan-wing system can be separated into its components as the lift produced by the fan, the lift produced by the wing, the fan induced wing lift and the lift induced due to the ducting of the propeller. The theoretical model calculates all the components taking into account the interference effects.

The interference of the wing on the ducted fan can be considered to be due to two effects. The first is due to the wing induced upwash in the duct. This has been found to be negligible as compared to the fan induced inflow at moderate and high values of C_F. The second is due to the turning of the flow by the wing. The flow on the wing surface is tangential to it and thus perpendicular to the fan axis of rotation.

For the interference effect of the fan on the wing the induced upwash on the wing due to the fan singularity system can be determined. This upwash in turn determines the wing singularity system. However, a simplified model for the effect of the fan on the wing has been suggested and tried here. It assumes that the effect on the wing due to the other

fan singularities is negligible as compared to the duct bound vorticity effect. Earlier work [24] which takes into account the deflection of the fan slipstream due to the forward velocity suggests that the effect of the deflection is negligible. Hence, in the present model, the curvature of the fan slipstream is neglected. The theoretically predicted fan induced wing lift has been found to be in agreement with the experiment [5].

In Chapter II, a vortex lattice model for the wing with a duct and the wing is discussed. The wing is replaced by a flat plate which in turn is replaced by a vortex lattice. Flow tangency condition is used to determine the strengths of the vortices in the lattice. For the wing with a duct, the same model is used except that no vortices are assumed to lie in the duct.

In Chapter III, the fan model is discussed. The fan has been replaced by a series of singularities representing the fan duct, fan disc, fan wake and the fan centre body. These singularities are determined by an iterative procedure and they are used to get the fan inflow and forces and moments on the fan.

Chapter IV discusses the proposed model for the fan-wing system and the various interference effects.

Chapter V gives the details of the example considered for comparing the theory with the experiment. The model is

a semi-span tapered wing with a span of 60 inches, root chord of 40 inches and a taper ratio of half. It has a single wing submerged lifting fan.

Chapter VI gives the results and the conclusions along with suggestions for further work and improvement on the suggested model.

The computer program is given in Appendix C.

CHAPTER II

WING MODEL

The wing is assumed to be thin and is replaced by a flat plate at an angle of attack. A vortex lattice representation is used for representing the flat plate. The lattice is made up of a series of horse shoe vortices, as shown in Figure 4. The arrangement is so chosen as to concentrate the vortices in regions of steep pressure gradients i.e. near the leading edge and near the hole. They are arranged in chordwise strips for ease of computation. For the case of the wing with a hole, no vortices are assumed to lie in the hole, Figure 5.

Having determined the geometry of the vortex lattice, it is necessary to determine the strengths of all the individual vortices. This is done by suitably choosing as many control points as the number of vortices and enforcing the flow tangency condition at each control point. This provides an unknown vortex strength and a known boundary condition for each of the horse-shoe vortices in the lattice.

1. MATHEMATICAL FORMULATION:

From Biot Savart law the induced velocity $d\underline{v}$ due to an element $d\underline{s}$ of a vortex of strength Γ at a point distant

$\underline{R}$ from the vortex element, is given by,

$$d\underline{v} = \frac{\Gamma}{4\pi} \frac{d\underline{s} \times \underline{R}}{|\underline{R}|^3} \quad . \qquad \ldots(2.1)$$

Where $d\underline{s}$ is the coordinate along the vortex such that looking along it the circulation is clockwise. For a vortex of length $l_1 l_2$, Figure 2, the magnitude of the induced velocity is,

$$|\underline{v}| = \frac{\Gamma}{4\pi a} (\cos \theta_1 + \cos \theta_2), \qquad \ldots(2.2)$$

the direction being given by $d\underline{s} \times i\underline{a}$.

A right handed cartesian coordinate systems with origin at the leading edge of the wing root chord, C_R, is used to represent the wing. The positive x axis lies along the wing chord pointing towards the wing trailing edge and the y axis is positive to the starboard side, Fig. 1. The horse-shoe vortex abcd of strength Γ induces a down-wash at the control point P lying in the plane of the vortex. The down-wash 'w' is taken positive in the negative Z direction. Now,

$$w = w_{ab} + w_{bc} + w_{cd}, \qquad \ldots(2.3)$$

where the subscript denotes the portion of the horse-shoe vortex abcd. Let,

$$\begin{aligned} X &= x_p - x_v \\ Y &= y_p - y_v \\ Y' &= y_p + y_v \;, \end{aligned} \qquad \ldots(2.4)$$

with x and y being non-dimensional quantities with respect to the root chord C_R. From Biot Savart law,

$$w_{ab} = \frac{\Gamma}{4\pi \; C_R \; (Y+s)} \left(\frac{X}{\sqrt{X^2+(Y+s)^2}} + 1 \right),$$

$$w_{bc} = \frac{\Gamma}{4\pi \; C_R \; X} \left(\frac{(Y+s)}{\sqrt{X^2+(Y+s)^2}} - \frac{(Y-s)}{\sqrt{X^2+(Y-s)^2}} \right),$$

$$w_{cd} = -\frac{\Gamma}{4\pi \; C_R \; (Y-s)} \left(\frac{X}{\sqrt{X^2+(Y-s)^2}} + 1 \right).$$

Hence,

$$w = \frac{\Gamma}{4\pi \; C_R} \; f(X,Y,s), \qquad \ldots(2.5)$$

where,

$$f(X,Y,s) = \frac{Y+s}{X\sqrt{X^2+(Y+s)^2}} + \frac{1}{(Y+s)} \left(1 + \frac{X}{\sqrt{X^2+(Y+s)^2}}\right)$$
$$- \frac{(Y-s)}{X\sqrt{X^2+(Y-s)^2}} - \frac{1}{(Y-s)} \left(1 + \frac{X}{\sqrt{X^2+(Y-s)^2}}\right) \qquad \ldots(2.6)$$

Similarly the downwash at P due to the image vortex is obtained by replacing Y by Y' in Eq. (2.5). Denoting f(X, Y',s) by f', the total downwash at P due to vortex abcd and its image is,

$$w = \frac{\Gamma}{4\pi \; C_R} \; F(X,Y,Y',s) \qquad \ldots(2.7)$$

where,

$$F = f + f'$$

~~The function F depends only on the geometry i.e. the~~

The function F depends only on the geometry i.e. the vortex span, location and the location of the control point. generalising the above argument for a lattice of 2n vortices and n control points, we have,

$$\{w\} = [F]\left\{\frac{\Gamma}{4\pi\ C_R}\right\}, \qquad \ldots(2.8)$$

where, the element F_{ij} of the matrix $|F|$, represents the influence of the j-th vortex on the i-th control point. For a number of vortices, at a given spanwise station, the sectional lift is,

$$L = \rho V\ \Sigma\Gamma\ ,$$

so that the section lift coefficient is,

$$C_l' = \frac{L}{\frac{1}{2}\ V^2\ C_R} = \frac{2}{VC_R}\ \Sigma\Gamma\ . \qquad \ldots(2.9)$$

Also,

$$C_l' = \Sigma(C_p\,\Delta x), \qquad \ldots(2.10)$$

where, C_p is the pressure coefficient assumed to be constant over a small fraction of the chord Δx and the summation is over the chord of the assumed spanwise station. It can be shown that for each vortex, circulation Γ can be related to a pressure coefficient C_p at the same chordwise position as the vortex centre, cf. Appendix A. Thus,

$$C_p = \frac{1}{\Delta x}\left(\frac{2\Gamma}{VC_R}\right) \qquad \ldots(2.11)$$

Now from Eq. (2.8), we have,

$$\frac{8\pi}{V}\{w\} = [F] \left\{\frac{2\Gamma}{VC_R}\right\},$$

or

$$\left\{\frac{2\Gamma}{VC_R}\right\} = 8\pi \; [F]^{-1} \left\{\frac{w}{V}\right\} \; . \qquad \ldots(2.12)$$

Multiplying both sides of the Eq. (2.12) by $[1/\Delta x]$ and using Eq. (2.11), we obtain,

$$\{C_p\} = \left[\frac{8\pi}{\Delta x}\right] [F]^{-1} \left\{\frac{w}{V}\right\} \; . \qquad \ldots(2.13)$$

Also the local moment coefficient can be obtained as,

$$\{x' \; C_p\} = [x'] \; \{C_p\} \qquad \ldots(2.14)$$

where, x_i' is the distance of the vortex centre from the leading edge of the wing or the distance from the line about which the moment coefficient has to be determined. Now to obtain the overall lift and moment coefficients, Eqs.(2.13) and (2.14) can be numerically integrated first in the chordwise and then in the spanwise directions. Thus,

$$C_L' = \int_{-b/C_R}^{b/C_R} \int_{x_{le}}^{x_{te}} \{C_p\} \; dxdy \qquad \ldots(2.15)$$

and,

$$C \qquad C_M' = \int_{-b/C_R}^{b/C_R} \int_{x_{le}}^{x_{te}} \{X' \; C_p\} \; dxdy \qquad \ldots(2.16)$$

To get the conventional C_L and C_M, C_L' should be multiplied by XC_R^2/S and C_M should be multiplied by $C_R^3/(S\bar{e}_A)$,

where,

$$C_M = \frac{M}{q_\infty S\bar{c}_A} .$$

While choosing the vortex network for the wing the following must be borne in mind. The control points should not lie on the bound or trailing portion of any vortex. The choice of Δx determines the vortex and control point locations. It should be so selected as to concentrate the bound vortices in the portions of steep pressure gradients e.g. leading edge and near the duct. For selecting Δx, the wing was divided into spanwise sections whose widths were chosen so as to concentrate more vortices near the duct. Each spanwise section was then divided into a number of rectangular sections whose length was small near the leading edge of the wing and large near the trailing edge. In each of these rectangular sections a bound vortex was placed at a quarter of the length of the section and the control point was placed at three quarters, from the leading edge of the section. The number of rectangular sections determines the number of vortices. The only limitation on the number of vortices is the ease of computation and the time available.

A computer program in Fortran IV has been made for the vortex lattice method, cf. Appendix C. The method used for numerical integration is Simpson's method with unequal interval, cf. Appendix B.

Since the theoretical pressure distribution for a flat plate is infinite at the leading edge a curve of the form $ax^{-1/2} + bx^{1/2} + cx^{3/2}$,[21] was fitted to the first three calculated points of the spanwise station, and integrated from the leading edge to the first point. For the remaining points, starting from the first point to the trailing edge, Simpson's method was used for integration.

CHAPTER III

FAN MODEL

A detailed singularity distribution is used for representing the fan because of the availability of a NASA Computer Program [18]. A source and a sink is used for representing the fan centre body. The propeller disc is replaced by a series of equal area annuli each having a constant strength bound vortex and a cylindrical shed vortex. The duct is replaced by a duct bound vorticity distributed from the leading edge to the trailing edge. The effect of the duct thickness is represented by a series of sources and sinks along the duct chord, Fig. 8.

For a given angle of attack α_f, advance ratio J and fan and duct configuration an iterative procedure is used to arrive at the fan input velocity profile, thrust, normal force and pitching moment coefficients of the fan. The method used treats the ducted fan at an angle of attack as ducted fan with axial flow and ducted fan with crossflow. The two solutions are superposed to obtain the final solution.

1. AXIAL FLOW:

For the ducted fan in axial flow the local blade performance is obtained by using blade element theory. The bound

circulation, Γ_w, in the w-th annulus is,

$$\frac{\Gamma_w}{RV} = \frac{1}{2} C_L \frac{b_w}{R} \frac{u_w}{V} \left[\left(\frac{\bar{r}_w \omega}{u_w}\right)^2 + 1\right]^{1/2} . \qquad \ldots(3.1)$$

Where, C_L is the blade sectional lift coefficient, b is the blade chord, R is the propeller radius, u is the induced axial velocity, $\bar{r}_w$ is the mean radius of the w-th annulus, V is the free stream velocity and ω is the angular velocity. Strength of the w-th internal trailing vortex cylinder is,

$$\frac{\gamma_w}{V} = \left[\left(1 + \sum_{m=w+1}^{z} \frac{\gamma_m}{V}\right)^2 + \frac{N}{\pi J'} \left(\frac{\Gamma_w}{RV} - \frac{\Gamma_{w+1}}{RV}\right)\right]^{1/2} - \left(1 + \sum_{m=w+1}^{z} \frac{\gamma_m}{V}\right) \qquad \ldots(3.2)$$

where, $J' = \frac{V}{R\omega}$, $\qquad \ldots(3.3)$

γ_m is the strength of the m-th shed vortex cylinder, N is the number of fan blades and z is the number of equal area annuli making up the fan disc area. The strength, γ_z, of the vortex cylinder shed from the duct trailing edge is,

$$\frac{\gamma_z}{V} = \left[1 + \frac{N}{\pi J'} \left(\frac{\Gamma_z}{RV}\right)\right]^{1/2} - 1 \qquad \ldots(3.4)$$

The Eqs. (3.1)-(3.4) are functions only of the fan blade geometry, advance ratio J and the fan inflow profile. The fan blade geometry and advance ratio are known. Thus knowing the inflow profile the above circulations can be determined. Now the inflow velocity to the fan, u_m, is given by the axial components of the velocity induced by all

the fan singularities and the free stream axial velocity,

$$\frac{u_m}{V} = 1 + \frac{u_{\gamma D}}{V} + \frac{u_{\gamma}}{V} + \frac{u_{qD}}{V} + \frac{u_{CB}}{V} + \frac{1}{2} \sum_{w=m}^{z-1} \frac{\gamma_w}{V}, \quad \ldots(3.5)$$

where, u denotes the axial component of the induced velocity and the subscripts γ_D, γ, q_D and CB denote the duct bound vorticity, $_z$, duct thickness and centre body respectively.

The free stream, trailing vortex cylinders and the centre body induce components of flow through the duct camber line. To cancel this flow and make the net flow tangential to the camber line a bound vorticity, γ_D, is placed on the duct. It is assumed to be in the form of a Glauert series.

$$\frac{\gamma_D}{\gamma} = C_o \operatorname{Cot} \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta \quad \ldots(3.6)$$

The C_n coefficients are found by the flow tangency condition on the duct reference cylinder. The duct reference cylinder is defined as that cylinder having the same radius as the duct trailing edge. Thus,

$$v_{\gamma_D} + v_{\gamma} + v_{\gamma_w} + v_{CB} = (V + u_{\gamma_D} + u_{\gamma} + u_{\gamma_w} + u_{CB}) \frac{dr_e}{dx}$$

$$\ldots(3.7)$$

where $\frac{dr_e}{dx}$ is the effective slope of the camber line and v denotes the radial component of induced velocity.

2. CROSS-FLOW:

The ducted fan in a cross flow is replaced by a thin cylindrical duct in a cross flow V sin α_f. A non-axisymmetric vorticity distribution, γ_α, is placed on the duct to

cancel the cross flow through the duct. Thus γ_α, the duct bound vorticity has the form,

$$\frac{\gamma_\alpha}{V} = \sin \alpha_f \cos \phi \left[C_0 \cot \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta \right] \quad \ldots(3.8)$$

In addition, there is a distribution of free trailing vortex filaments because of the variation of bound vortex strength around the circumference of the duct. The effect of the angle of attack on the fan loading is not included in the computer program because of relatively small contribution, [18]

3. ITERATIVE CALCULATION PROCEDURE:

An initial inflow profile of 2V is assumed and the blade element calculations are made to obtain the fan and the fan-wake characteristics. The flow tangency condition is then applied to determine the duct bound vorticity. Having found all the singularity distributions the fan inflow profile is calculated. This is compared with the previous inflow profile. If the two are not the same, then a new inflow, mean of the two, is assumed and the procedure repeated. A series of iterations are gone through till the flow profile converges to within the desired accuracy. The forces and moments on the duct and the fan are then calculated using the final values of the axisymmetric and nonaxisymmetric singularity distributions.

CHAPTER IV

COMBINING FAN-WING MODEL

The combined fan-wing system is replaced by a vortex lattice for the wing portion and a set of singularities for the ducted fan portion and its wake. The wing singularity system is the same as the singularity system for a wing with a duct , cf. Chapter II. The ducted fan singularity system is the same as given in Chapter III.

The total lift produced by the fan-wing system is made-up of the lift produced by the wing, the fan induced wing lift and the lift of the fan in the presence of the wing. The last mentioned lift component consists of the lift of the fan and the lift induced due to ducting of the fan.

The method used for solving the problem, considers each of the lift components separately and sums them up to get the total lift produced by the fan-wing system. The strength of the wing singularities and hence the forces are determined by using the flow tangency condition on the surface of the flat plate used for representing the wing. For the forces on the fan in the presence of the wing, the ducted fan is considered together with the interference of the wing on the fan. The wing interference effect on the fan is considered due to two effects. The first is the wing induced upwash in the duct. This has been found to be negligible as compared to the fan

induced inflow at moderate and high values of C_F. The second is due to the turning of the flow by the wing. The flow on the wing surface is tangential to it and thus perpendicular to the fan axis of rotation. Thus for the ducted fan in the presence of the wing, it is sufficient to consider the fan at approximately ninety degrees incidence with respect to the fan axis of rotation. For this incidence and given advance ratio the fan program determines the strengths of the various singularities of the ducted fan and the resulting inflow, forces and moments.

For the fan induced wing lift, the induced upwash on the wing due to the ducted fan singularities is first determined and the corresponding wing singularity system found. This is the interference effect of the fan on the wing. It is considered in more detail in subsection 1.

Now the strength of the singularity system representing the fan-wing system is completely determined. The corresponding velocities, pressures and forces can be computed.

1. EFFECT OF THE DUCTED FAN ON THE WING:

The singularities used for representing the ducted fan and its wake are: source and sink distribution along the duct chord representing the duct thickness, axisymmetric duct bound vorticity due to the axial flow component of the free stream, non-axisymmetric duct bound vorticity due to the cross-flow component of the free stream, trailing vortices

due to variation of the duct bound vorticity along the circumference, the propeller bound vorticity representing the propeller, the shed vortex cylinders representing the propeller wake, and a point source and a point sink representing the propeller centre body. All these singularities influence the flow on the wing. Their influences are derived separately and then super-imposed to get the effect of the ducted fan on the wing.

EFFECT OF DUCT THICKNESS:

The duct is assumed to be thin and as such the effect of its thickness on the wing is neglected as a first approximation.

INDUCED VELOCITY DUE TO THE AXISYMMETRIC DUCT BOUND VORTICITY:

The induced velocity at a point P(x, y, z) or (x, r, $\emptyset$), Fig. 13, due to a vortex ring of strength Γ, is given by v_x the axial component and v_r the radial component, [26].

$$v_x(x,r) = -\frac{\Gamma}{2\pi r'} \frac{1}{\sqrt{x^2+(r+1)^2}} \left\{F(k) - \left[1 + \frac{2(r-1)}{x^2+(r-1)^2}\right] E(k)\right\}, \qquad \ldots (4.1)$$

and,

$$v_r(x,r) = \frac{\Gamma}{2\pi r'} \frac{x}{r\sqrt{x^2+(r+1)^2}} \left\{F(k) - \left[1 + \frac{2r}{x^2+(r-1)^2}\right] E(k)\right\}, \qquad \ldots (4.2)$$

where, r' is radius of the vortex ring, $x = x/r'$, $r = r/r'$, $F(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind respectively and

$$k^2 = \frac{4r}{x^2 + (r+1)^2} \quad . \qquad \ldots(4.3)$$

Now the duct bound vorticity is represented as a cylindrical vortex sheet with vorticity varying continuously from the leading edge to the trailing edge of the duct. Let γ_D be the circulation per unit length of the duct. From [18],

$$\gamma_D = \gamma_z \left(C_o \cot \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\right), \qquad \ldots(4.4)$$

where, $\theta = \cos \left(\frac{2x'}{c}\right)$

Thus the strength of the elemental vortex ring, Fig. 14,

$$\Gamma = \gamma_D (x')\, dx'$$

$$= \gamma_z \left(C_o \operatorname{Cot} \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\right) dx' \qquad \ldots(4.5)$$

putting, $X = x/r'$, $X' = x'/r'$, $r = r/r'$

$$= r' \gamma_z \left(C_o \operatorname{Cot} \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\right) dX'$$

From Eq. (4.1),

$$dv_x(X,r) = -\frac{r'\gamma_D\, dX'}{2\pi r'} \frac{1}{\sqrt{(X-X')^2+(r+1)^2}}$$

$$\left\{F(k') - \left[1 + \frac{2(r-1)}{(X-X')^2+(r-1)^2}\right] E(k')\right\}$$

where,

$$k'^2 = \frac{4r}{(X-X')^2+(r+1)^2}, \qquad \ldots(4.6)$$

$$\text{or} \quad v_x(0,r) = -\int_{-c/2r'}^{c/2r'} \frac{1}{2\pi} \gamma_z \left(C_0 \operatorname{Cot} \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\right) \frac{1}{\sqrt{X'^2+(r+1)^2}} \left(F(k') - \left[1 + \frac{2(r-1)}{(X')^2+(r-1)^2}\right] E(k')\right) dX' \quad , \qquad \ldots(4.7)$$

$$\text{and,} \quad v_r(0,r) = \int_{-c/2r'}^{c/2r'} \frac{1}{2\pi} \gamma_z \left(C_0 \operatorname{Cot} \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\right) \frac{X'}{r\sqrt{X'^2+(r+1)^2}} \left(F(k') - \left[1 + \frac{2r}{X'^2+(r-1)^2}\right] E(k')\right) dX' \quad . \qquad \ldots(4.8)$$

It should be noted that v_x is positive in the positive x direction, which is the z direction in the coordinate system used for representing the wing. Also note that downwash is taken positive in the wing coordinate system.

INDUCED VELOCITY DUE TO THE NON AXISYMMETRIC DUCT BOUND VORTICITY

The duct bound vorticity, per unit length of the duct, due to the angle of attack of the duct is given by [18],

$$\gamma_\alpha = V \sin \alpha_f \cos \phi' \left[C_0 \operatorname{Cot} \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\right] \quad , \qquad \ldots(4.9)$$

where $\theta = \cos^{-1}\left(\frac{2x'}{c}\right)$, α_f is the angle of attack of the fan, and ϕ and ϕ' are shown in Fig. 13.

For a single vortex ring of strength Γ [26], the induced axial velocity is,

$$v_x(X,\ r) = -\int_0^{2\pi} \frac{\Gamma}{4\pi r'} \frac{r\cos(\phi-\phi') - 1}{|X^2+r^2+1-2r\cos(\phi-\phi')|^{3/2}}\, d\phi' , \qquad \ldots(4.10)$$

refer Fig. 13.

Now for an elemental vortex ring,

$$\Gamma = \gamma_\alpha\, r'\, dX'$$

and the induced axial velocity $v_{\alpha x}$ is,

$$v_{\alpha x}(X,\ \phi,\ r) = \int_{-c/2r'}^{c/2r'} \int_{o}^{2\pi} \frac{\gamma_\alpha}{4\pi} \frac{r\cos(\phi-\phi')-1}{((X-X')^2 + r^2+1-2r\cos(\phi-\phi'))^{3/2}}\, d\phi' dX' , \qquad \ldots(4.11)$$

refer Fig. 14.

Or the influence of the duct bound vorticity due to angle of attack, on the wing is,

$$v_{\alpha x}(0,\ \phi,\ r) = \frac{V\sin\alpha}{4\pi} \int_{-c/2r'}^{c/2r'} \Big(C_0 \operatorname{Cot}\frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta\Big) \int_0^{2\pi} \cos\phi' \frac{(r\cos(\phi-\phi')-1)}{((X-X')^2+r^2+1-2r\cos(\phi-\phi'))^{3/2}}\, d\phi'\, dX' , \qquad \ldots(4.12)$$

Similarly, the radial velocity component due to a single vortex of strength Γ, [26], refer Fig. 13,

$$v_r = \int_0^{2\pi} \frac{\Gamma}{4\pi r'} \frac{X \cos \phi'}{(X^2+r^2+1-2r \cos (\phi-\phi'))^{3/2}} \, d\phi' \qquad \ldots(4.13)$$

Thus the induced radial velocity component of the duct bound vorticity due to angle of attack, $v_{\alpha r}$, on the wing is,

$$v_{\alpha r}(0, \phi, r) = \frac{V \sin \alpha}{4\pi} \int_{-c/2r'}^{c/2r'} X'(C_0 \cot \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta) \int_0^{2\pi} \cos \phi' \, (X'^2 + r^2 + 1-2r \cos (\phi-\phi'))^{-3/2} \, d\phi' \, dX' \qquad \ldots(4.14)$$

The integrals involved were integrated numerically to get the influence of the duct on the wing.

INDUCED VELOCITIES DUE TO THE TRAILING VORTICES AND PROPELLER BOUND VORTICITY:

As a first approximation the effect of the trailing vortex filaments, due to the variation of duct bound vorticity around the duct circumference, and the propeller bound vorticity on the wing has been neglected.

INDUCED VELOCITIES DUE TO THE PROPELLER SHED VORTEX CYLINDERS:

The shed vortex cylinders are assumed to be straight and extending to infinity. As such their effect is confined to the area within the cylinder and have no influence on the wing

flow field. However at higher forward velocities the shed vortex cylinders tend to bend downstream and they induce a flow field on the lower surface of the wing. Murti |24| considers the effect of the deflected fan jet and finds its influence to be very small. Hence in the present model the curvature of the fan jet is not considered.

INDUCED VELOCITIES DUE TO THE CENTRE BODY:

The model assumed for the centre body, in the fan program, is a Rankine body. Given the dimensions of a body of revolution (ξ_o and h), Fig. 11, it can be approximated by a source and a sink whose locations are given by,

$$h^2 \sqrt{a^2 + h^2} = \frac{(\xi_o^2 - a^2)^2}{o} \qquad \ldots(4.15)$$

Induced velocity at a point (ξ , r) is given by,

$$\frac{u(\xi, r)}{V} = \frac{(\xi_o^2 - a^2)^2}{4\,\xi_o\, a} \cdot \left\{ \frac{(\xi + a)}{[(\xi + a)^2 + r^2]^{3/2}} - \frac{(\xi - a)}{[(\xi - a)^2 + r^2]^{3/2}} \right\} \qquad \ldots(4.16)$$

$$\frac{v(\xi, r)}{V} = \frac{(\xi_o^2 - a^2)}{4\,\xi_o\, a} \left\{ \frac{r}{[(\xi + a)^2 + r^2]^{3/2}} - \frac{r}{|(\xi - a)^2 + r^2|^{3/2}} \right\} \qquad \ldots(4.16)$$

where u and v are the axial and radial components. However, for the example considered there is no centre body and hence the induced velocity due to the centre body is not taken into account.

From the above discussion, it will be seen that in-effect only the duct bound vorticity is considered for getting the effect of the fan on the wing. The effect of other singularities is assumed to be negligible as compared to the duct bound vorticity. The deflection of the fan slip stream with forward velocity is also neglected and so the fan slip stream is semi infinite and nearly perpendicular to the free stream for the assumed fan-wing model.

CHAPTER V

EXAMPLE

As an example we consider a semi-span trapizoidal wing model, Fig. 3, with a submerged fan, which has been treated experimentally elsewhere [5]. The details of the model are given below.

WING:

The wing is a semi-span trapizoidal wing of aspect ratio four, semi-span area of 12.5 square feet, taper ratio of 0.5 and a 0^{o} sweep of the half-chord line. The airfoil section of the wing is NACA 16-015 (employed to give nearly equal fore and aft duct length). The aerodynamic axis of the fan is on the mean aerodynamic chord at 0.5 $\bar{c}_A$.

FAN:

The fan has four blades. The blade form characteristics are given in Fig. 7. The airfoil section of the propeller blade~~xx~~is NACA 16-309 on a 66 series mean line. The clearance between the propeller tip and the duct is 0.006 times propeller radius.

For the purpose of mathematical simulation, the semi-span wing is replaced by a vortex lattice of 64 horse-shoe vortices as shown in Fig. 4. For simulating the wing with a

vortex lattice of 48 horse-shoe vortices as shown in Fig. 5. Figs. 4 and 5 also show the control points location on the wing. Table 2 gives the details of the vortices and the control points for the wing and Table 3 gives the details for the wing with a duct, cf. Appendix C. Fig. 8 gives a schematic view of the singularities used for representing the fan. The details are presented in input data, cf. Appendix C and Table 1.

CHAPTER VI

RESULTS AND CONCLUSIONS

The experimental results given by Hickey and Ellis [5] have been used here for comparison with the theory. It should be noted that wind tunnel corrections have not been incorporated in the experimental results given in [5]. Some corrections can be expected because at high propeller loadings as much as ten percent of the wind tunnel airflow passes through the propeller duct. There is a blockage effect due to this wake, the propeller drive motor and the supporting struts which are also exposed to the wind tunnel flow. Wind tunnel boundary corrections were also not considered.

The vortex lattice method used for the simulation of the wing predicted higher values for the wing lift coefficient and the lift curve slope as compared to the experiment, Fig. 17. From the experimental curves, it is seen that a wing with a duct gives some lift even at zero incidence unlike the wing without the duct. As the theoretical model replaces the wing with a flat plate no such effect can be expected. The theory also does not take into account any duct induced separation on the wing upper surface. The wing with a duct corresponds to the case of the wing with a fan, fan not working or working with very low values of C_F. This flight region of the fan-wing system is not of interest. Our interest lies in the transition regime.

The predictions of the fan program for the propeller thrust agree very well with the experiment at low rpm, but at higher rpm it predicts higher values for the propeller thrust, Fig. 18. It is not justifiable to comment on the theoretical predictions at high rpm as then the experimental results are amenable to large wind tunnel interference and blockage corrections. The variation of propeller thrust with q_∞ for a given rpm is small for both theory and experiment. However, for the range of q_∞ considered the variations in the theoretical values are negligible.

For a given experimental value of C_F and q_∞, J can be found. For this J the theoretical C_F is found to be different from the experimental C_F. This is because the theory predicts higher thrust than the experiment at the same rpm. Thus for comparison of the results it is a choice between J and C_F. The theoretical and experimental comparisons are made at the same value of C_F. This essentially means that the theoretical and experimental propeller thrusts are the same. Thus in this comparison, the fact that the propeller gives higher thrust at high rpm has been circumvented. Fig. 20 compares the theoretical and experimental incremental forces due to the fan in the wing. At zero incidence the lift produced by the wing is made up of the fan induced wing lift and the lift due to the ducting of the propeller. From the figure we see that except at very low and very high values of C_F, the predictions of the theoretical model are very encouraging. This high-lights

the fact that for the fan-wing system the duct has a significant effect on the fan induced lift.

Figure 19 compares the total lift produced by the fan-wing system at nearly same values of C_F. The model predicts comparable results except at the higher value of C_F.

The experimental results do not give the fan induced upwash distribution and the pressure distribution over the wing. Fig. 21 gives the chordwise fan induced upwash distribution on the wing at three spanwise stations. Fig. 22 gives the chordwise overall pressure distribution at the same three spanwise stations on the wing. As expected the effect of the fan has been found to be considerable near the fan but small away from it. Fig. 23 gives the variation of $C_{M_{le}}$ with C_L.

It is suggested that in future work, the effects of the fan singularities, which have not been considered, should be taken into account for obtaining the effect of the fan on the wing. Further detailed in flight experimental investigations should be carried out on a model or the prototype and the theoretical predictions compared against the results obtained. Experiments devised with the minimum of wind tunnel interference or determinable tunnel interference will also be very useful for comparison.

REFERENCES

1. Vanderlip Edward, G. Schneider John, J. — Design and Development of the Shaft Driven Fan-in-Wing VTOL Aircraft. Ann. N.Y. Acad. Sc. Vol. 107, Art. 1, pp. 177-195 (1963).

2. Asmus, F.J. — Design and Development of Tip Turbine Lift Fan. Ann N.Y. Acad. Sc. Vol. 107, Art. 1, pp. 147-176, (1963).

3. Wardlaw, R.L. McEachern, N.V. — A Wing Submerged Lifting Fan Wind Tunnel Investigations and Analysis of Transition Performance. NRC, NAE LR-243 (April 1959).

4. Duvivier, J.F. McCallum, R.B. — Investigation of Tilting Duct and Fan-Wing in Transition Flight. U.S. Army, TREC, TR-61-19 (Dec. 1960)

5. Hickey, D.H. Ellis, D.R. — Wind Tunnel Tests of a Semi Span Wing with a Fan Rotating in the Plane of the Wing. NASA TN D-88 (Oct. 1959).

6. Hackett, J.E. — Wind Tunnel Tests on a Streamlined Fan Lift Nacelle. ARC, R and M - 3470 (Oct. 1965).

7. Hickey, D.H. Hall, L.P. — Aerodynamic Characteristics of a Large Scale Model with Two High Disc. Loading Fans Mounted in the Wing. NASA TN D-1650 (Feb. 1963).

8. Kirk, J.V. Hickey, D.H. Hall, L.P. — Aerodynamic characteristics of a Full Scale Fan in Wing Model Including Results in Ground Effect with Nose Pitch Control. NASA, TN D-2368 (July, 1964).

9. Chambers, J.R. — Static and Dynamic Longitudinal Stability Derivatives of a Powe 0.18 Scale Model of a Fan-in-W VTOL Aircraft. NASA TN D-4322 (1968).

10. Spreeman, K.P. — Induced interference Effects on Jet and Burried Fan Configurations in Transition. NASA TN D-731 (1961).

11. Kuhn, R.E. — Review of Basic Principles of V/STOL Aerodynamics. NASA, TN D-731 (1961).

12. Kirby, R.H. — Aerodynamic Characteristics of Propeller Driven VTOL Aircraft. NASA, TN D-730 (1961).

13. Trebble, W.J.G. — Wind Tunnel Experiments on a Lifting Jet Body with and Without Wings. ARC CP-718 (1963).

14. Wyatt, L.A. — Preliminary Note on a Wind Tunnel Test of a Wing Fitted with Multiple Lifting Fans. ARC 21-377 (Oct. 1959).

15. Wyatt, L.A. — Unpublished M.O.A. work. (Cross Reference from 13).

16. Goldsmith, R.H., Hickey, D.H. — Characteristics of Aircraft with Lifting Fan Propulsion Systems for V/STOL. IAS Paper No. 63-27 presented at the IAS 31st Annual Meeting, New York, N.Y. (Jan. 1963).

17. Mendenhall, M.R., Spangler, S.B. — Theoretical Study of Ducted Fan Performance. NASA CR-1494 (Jan. 1970).

18. Mendenhall, M.R., Spangler, S.B. — A Computer Program for the Prediction of Ducted Fan Performance. NASA CR-1495 (Feb. 1970).

19. Norland, S.A. — An Investigation of the Lift Produced by a Fan in a Two Dimensional Wing. MIT SM Thesis.

20. Martin Eugene, E. — The Lift Produced by a Wing Mounted Lift Fan at Flight Speeds Below Transition. J. Am. Helicopter Soc. 9 (Apr. 1964).

21. McEachern, N.V. Currie, M.M. — A Vortex Lattice Lifting Surface Theory for Wings with Submerged Fans. NRC, NAE LR-311 (Aug. 1961).

22. Monical, R.E. — A Method of Representing Fan-Wing Combinations for Three Dimensional Potential Flow Solutions. J. of Aircraft, Vol. 2, No. 6, pp. 527-530 (Dec. 1965).

23. Williams, J. Wood, M.N. — Aerodynamic Interference Effects with Jet Lift V/STOL Aircraft Under Static and Forward Speed Conditions. Flugwiss, 15.7, pp.237-257 (July 1967).

24. Murti, C.V.R. — Effect of Submerged Lift Fans on the Circulation of Wings. IIT Kanpur, M. Tech. Thesis (May, 1969).

25. Robinson, A. Laurmann, J.A. — Wing Theory, Cambridge University Press (1953).

26. Khuchemann, D. — Aerodynamics of Propulsion, McGraw Hill (1953).

27. Byrd, P.F. Friedman, M.D. — Handbook of Elliptic Integrals for Engineers and Scientists. 2nd Edition. Springer-Verlag (1971).

28. Salvadori, M.G. Baron, M.L. — Numerical Methods in Engineering 2nd Edition. Prentice Hall India (1966).

APPENDIX A

RELATION BETWEEN VORTEX STRENGTH AND COEFFICIENT OF PRESSURE:

It can be shown that for each vortex, the circulation can be related to the pressure coefficient at the same chordwise position as the vortex centre. Take a vanishingly thin element of area of length Δx surrounding a typical vortex, cf. Fig. 12. Constant C_p over Δx is equivalent to the assumptions that the induced velocity u over Δx can be replaced by a mean velocity $\bar{u}$ where $\bar{u} = \frac{1}{\Delta x} \int u dx$. If w_u and w_d are the downwash at the upstream and downstream ends, we have,

$$\frac{\Gamma}{C_R} = \oint \left(u \frac{dx}{C_R} + w \frac{dz}{C_R}\right)$$

$$= 2\bar{u} \frac{\Delta x}{C_R} + (w_d - w_u) \frac{\Delta z}{C_R}$$

$$2\bar{u} \frac{\Delta x}{C_R} \qquad \ldots (A-1)$$

as u and w are of the same order and Δz can be made negligibly small with respect to Δx. Applying Bernoulli's Eq. to the lower and upper surfaces gives,

$$\frac{1}{2} \rho V^2 + p_o = \frac{1}{2} \rho (V+\bar{u})^2 + p_u = \frac{1}{2} \rho (V-\bar{u})^2 + p_l$$

or
$$p_l - p_u = \frac{1}{2} \rho (4V\bar{u}) \qquad \ldots (A-2)$$

Defining,

$$C_{p_u} = \frac{p_u - p_o}{\frac{1}{2} \rho V^2} \;, \quad C_{p_l} = \frac{p_l - p_o}{\frac{1}{2} \rho V^2} \quad \text{and}$$

$C_p = C_{p_l} - C_{p_u}$, we obtain for C_p,

$$C_p = 4 \; (\breve{u}/V) \qquad \ldots(A\text{-}3)$$

Substituting for $\breve{u}$ in terms of Γ we obtain,

$$C_p = \frac{1}{\frac{\Delta x}{C_R}} \; \left(\frac{2\,\Gamma}{VC_R} \right) \qquad \ldots.(A\text{-}4)$$

Thus subject to the above assumptions every term in Eq.(2.9) can be equated to corresponding term in Eq. (2.10).

APPENDIX B

SIMPSONS METHOD WITH UNEQUAL INTERVALS:

The formula of Simpson's method with unequal intervals for numerical evaluation of an integral, $\int_{x_0}^{x_2} f(x)\,dx$, is derived below.

Take a two dimensional rectangular cartesian coordinate system. Let x_o, x_1 and x_2 be three arbitrary points, on the x-axis such that $x_2 > x_1 > x_o$. Let the values of the function $y = f(x)$ at these points be y_o, y_1 and y_2. Shift the origin to x_1 such that,

$$
\begin{aligned}
x_1 &= 0, \\
x_o &= -\alpha_1 H, \qquad \ldots\text{(B-1)} \\
x_2 &= \alpha_2 H.
\end{aligned}
$$

Let the function $y = f(x)$ be approximated by a quadratic,

$$y = Ax^2 + Bx + C$$

in the interval $x_o\ x_2$. Therefore,

$$
\begin{aligned}
y_o &= A(\alpha_1 H)^2 - B(\alpha_1 H) + C, \\
y_1 &= C, \qquad \ldots\text{(B-2)} \\
y_2 &= A(\alpha_2 H)^2 + B(\alpha_2 H) + C,
\end{aligned}
$$

where,

$$
\begin{aligned}
A &= \frac{y_o\,\alpha_2 + y_2\,\alpha_1 - y_1\,(\alpha_1 + \alpha_2)}{\alpha_1 \alpha_2 H^2\,(\alpha_1 + \alpha_2)} \\
B &= \frac{y_2 \alpha_1^2 - y_o\,\alpha_2^2 - y_1\,(\alpha_1^2 - \alpha_2^2)}{\alpha_1 \alpha_2 H\,(\alpha_1 + \alpha_2)} \qquad \ldots\text{(B-3)} \\
C &= y_1.
\end{aligned}
$$

Then the integral of f(x) in the interval (x_0, x_2) can be expressed as,

$$\int_{x_0}^{x_2} f(x)\,dx \simeq \int_{-\alpha_1 H}^{\alpha_2 H} (Ax^2 + Bx + C)\,dx$$

$$= \frac{A}{3} H^3 (\alpha_2^3 + \alpha_1^3) + \frac{B}{2} H^2 (\alpha_2^2 - \alpha_1^2)$$

$$+ CH (\alpha_2 + \alpha_1) \qquad \ldots\text{(B-4)}$$

substituting for A, B and C and simplifying,

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{H}{3\alpha_1\alpha_2} \Big[y_0\, \alpha_2 (\alpha_1^2 + \frac{1}{2}\alpha_1\alpha_2 - \frac{1}{2}\alpha_2^2)$$

$$+ \frac{y_1}{2}(\alpha_1 + \alpha_2)^3 + y_2(\alpha_1^3 - \alpha_1^2\alpha_2 + \alpha_2^2\alpha_1$$

$$+ \frac{3}{2}\alpha_1\alpha_2 - \frac{3}{2}\alpha_1^2) \Big] \qquad \ldots\text{(B-5)}$$

This is the desired formula. For using the above method the function value must be given at an odd number of points greater than three.

APPENDIX C

PROGRAM DESCRIPTION:

The program has been written in Fortran IV for IBM 7044 computer. It has been divided into two parts and if it is run as indicated, it does not require any other tapes except the normal system tapes. The reason for dividing the program into two separate parts is computer memory considerations and convenience of using the system as it exists in the Computer Centre at IIT Kanpur. However, it can be run as a single program if a scratch tape is provided.

The computation proceeds as follows. For a given fan in wing combination Part I of the program calculates the values of the various fan singularities, the thrust developed and the fan inflow velocities. Part II uses some of the results obtained in the first part to calculate the wing singularities in the presence of the fan and gives the overall pressure and forces on the wing. Part II can also be used to calculate the wing singularities both for a wing with or without a duct in the absence of the fan.

PART I DESCRIPTION:

Part I of the program is the NASA program, Ref. [18] with only slight modifications in the main program so that the strength GAM of the vortex cylinder trailing from the duct trailing edge and the coefficients C_n of the Glauert series

representing the duct bound vorticity, are also printed out. These are used as part of the input to Part II.

This consists of a main program and twenty subroutines. The relationship between the main program and the subroutines is given in Fig. 15. The subroutines used and their functions are given below.

MAIN controls the flow of program, calculates the local blade element performance, calculates the total fan-in-flow, compares the initial and computed inflows for convergence and calculates the new inflow profile if convergence is not achieved. It is in two parts - the initialisation part and the iteration and convergence controlling part.

INPUT determines the quantities used in the calculation that are functions only of the duct configuration, centre body geometry fan blade characteristics, number of fan annuli. This includes fourier series coefficients for velocities and vorticities which are functions only of duct chord to diameter ratio or duct thickness. It also reads the input data, checks it for certain specific errors and prints it out.

PKL does a table look up of the P_{kl} coefficient for various values of the duct chord to diameter ratio. The influence coefficients, P_{kl} are used in computing the radial velocity induced by a distribution of vortex rings. These are functions only of the duct chord to diameter ratio.

ELLIPS does a table look up for the complete elliptic integrals of the first and second kind given the arguments of the elliptic integrals.

LAMBDA does a table look up of Heuman Lambda Function given the two arguments of the function..

HUB calculates the location of the point source and the point sink used for representing the centre body model. It also computes the nondimensional axial inflow velocity induced at the mean radii of the fan annuli by the source sink distribution.

CAMBER calculates the induced camber coefficients combined with the geometric camber of the duct to give the duct effective camber.

PROP divides the fan annulus into the required number of annuli, computes the inner, outer and mean radii of the annuli and interpolates in the blade characteristics from the input table to get their values at the mean annuli radii.

CLALF does a table look up of the local blade lift coefficients taking into account the fact that after blade stall the blade lift coefficient is assumed to be $C_{l\ max}$.

ARCSIN computes the principal value of the angle given the value of the sine of the angle.

MATRIX performs matrix inversion and prints error messages if the matrix is singular.

FOURCS does a Fourier analysis of a function given a table of the function values. It computes an n term ($n \leq 50$) cosine series to fit the given function. However, only the first six terms of the series are used for the calculations in the program.

SRCRNG computes the non-dimensional axial inflow velocity, u/V, induced at the mean radii of the fan annuli by the source rings representing the duct thickness distribution.

VTXRNG computes the non-dimensional induced axial velocity to the fan due to the axi-symmetric duct bound vorticity γ_D.

ALFRNG calculates the non-dimensional induced axial velocity to the fan due to the non-axisymmetric duct bound vorticity γ_α.

GAMCYL gives the non-dimensional axial inflow velocity to the fan induced by the vortex cylinder trailing from the duct trailing edge.

BNCOEF calculates the Fourier series coefficients for the radial velocity induced along the duct reference cylinder by the vortex cylinder trailing from the duct trailing edge.

ANCOEF calculates the Fourier series coefficients for the radial and axial velocities induced along the duct reference cylinder due to shed vortex cylinders and the vortex cylinder trailing from the duct trailing edge.

CNCOEF applies the boundary condition of no flow through the duct camber line to get the Glauert series coefficients representing the duct bound vorticity.

PRESS computes the velocity induced on the duct reference cylinder by all the singularity distributions and then calculates the duct surface pressure coefficients.

OUTPUT computes the force and moment coefficients and prints out all the output of the program.

1. INPUT DESCRIPTION:

This section gives the description of the form in which the input to Part I is to be given. It gives the card number, the variable name in order of punching and the format in which the various duct and fan parameters are to be punched.

CARD NO. 1 contains any alphabetic and numeric information that is desired for identification purposes. This information is printed on the top of each page of output.

CARD NO. 2 contains the duct geometry information. The variables to be punched are duct chord-to-diameter ratio (C/D), the axial location of the fan within the duct (X_p/C), the maximum thickness to chord ratio of the airfoil section of the duct (t/C), the ratio of the duct trailing edge radius to the fan radius (R/R_p), the ratio of the centre body radius at the fan station to the fan tip radius (R_{CB}/R_p) and the convergence criterion (ϵ). The format for all the variables in this card is F 10.4.

CARD NO. 3 contains the four Fourier coefficients (R_n) of a cosine series fit to the duct camberline. If the duct is uncambered the four values are all zero. The format for all the four coefficients is F 10.4.

CARD NO. 4 contains the centre body geometry information. The variables to be punched are the centre body length as a fraction of the duct chord (l_{CB}/C), the location of the centre body nose in the duct coordinate system (X_{CB}/C). If the centre body extends forward of the duct leading edge then X_{CB}/C is negative. Ratio of the maximum radius of the centre body to its length (r_{max}/l_{CB}) and the x location of r_{max} ($X_{r_{max}}$) in the duct coordinate system are punched next. All the variables are punched in F 10.4 format.

CARD NO. 5 contains six quantities. Number of fan blades (NBLD), number of equal area annuli (NZ), into which the fan annulus is divided, the number of input stations in the fan blade characteristics (NZP), the number of x stations in which the duct surface pressure distribution is to be calculated (IR), an index (NPRES) which controls the form of the surface pressure coefficient and an index (NPRINT) which controls the quantity of output. All these are punched in the format I 5.

CARD NO. 6 contains the fan radii (r/R_p) at which the blade characteristics are to be input. There are NZP entries in this table and the format in which the values are punched is F 10.4. If NZP > 8 then the remaining values are punched on extra cards. The values must be put in the order of increasing radius ratio.

CARD NO. 7 contains the values of the fan blade chord (b/R_p) at the radial stations corresponding to the values in Card No. 6. The format for all the values is F 10.4.

CARD NO. 8 contains the fan blade pitch angle (β) in degrees at the radial stations corresponding to the values given in Card No. 6. The format for all the values is F 10.4.

CARD NO. 9 contains the fan blade thickness to chord ratio at the above mentioned radial stations, the format being F 10.4.

CARD NO. 10 contains the values of x/C at which the duct surface pressure coefficients are to be calculated. This card contains IR values. The values should be put in the ascending order with the limitation $0 \leq x/C \leq 1.0$. The format is F 10.4.

CARD NO. 11 is the last card required for a particular configuration. The first value is the run identification no. (NRUN), next is the index (NPHI) which is the number of azimuth station around the duct at which the surface pressure coefficients are to be calculated. Advance ratio (J) is given next. It is followed by the angle of attack (α_f) in degrees and the remaining five fields contain the values of the azimuth angles ϕ in degrees. The formats for the variables are I 5, I 5, F 10.4, F 10.4 and 5 F 10.4 respectively.

If runs are to be stacked then the following procedure should be used. If the same duct fan configuration is to be investigated for different J, α and ϕ then a new card with the same format as Card No. 11 is punched and placed after Card No. 11. This can be repeated for as many runs as desired.

If a new configuration is to be investigated then a blank card should follow Card No. 11 and it should be followed by the deck of the new case consisting of Card Nos. 1 through 11. The input to Part I for the fan of Reference [5] is given in Table 1.

2. LIMITATIONS ON THE INPUT DATA TO PART I:

In Card No. 2 the admissible range of C/D is $0 < C/D < 2.5$. The upper limit is an approximate upper limit because the P_{kl} coefficients built into the program have to be extrapolated for C/D greater than 2. As P_{kl} do not vary linearly with C/D an arbitrary limit of C/D = 2.5 is fixed beyond which extrapolation is meaningless. The admissible range for the duct t/C is $0.06 \leq t/C \leq 0.24$. This is because of the correction factors used in the calculation of duct pressure distribution. If t/C is outside this range then the pressure distribution will be slightly in error but all other calculations will be correct. The suggested range for ϵ is $0.001 \leq \epsilon \leq 0.01$. The iteration will stop when the profiles agree to within ϵ x 100 percent.

The limitations on the variables given in Card No. 5 are that NBLD should be greater than zero. The range for NZ is $2 \leq NZ \leq 24$. The range for NZP is $2 \leq NZP \leq 24$. IR must lie in the range $1 \leq IR \leq 25$. If the duct surface pressure is not required then also IR should lie in the indicated range. If NPRES = 1, then the surface pressure coefficient is based on the free stream dynamic pressure, if NPRES = 2 then it is

based on the propeller tip speed. For normal runs NPRINT = 0. If NPRINT = 1, an extra page of output is developed which contains a table of the components of the inflow to the propeller. If NPRINT = 10 then a page listing the Fourier cosine series coefficients for the u and v velocities induced at the duct reference cylinder is printed. If NPRINT = 11 then both the above optional outputs are printed.

The first value of the r/R_p punched in card No. 6 should be that corresponding to the hub and the last value should be approximately 1.

As there is a discontinuity in the pressure across the fan thus it is desirable to have the x/C values, punched in Card No. 10, to be those corresponding to the points ahead and behind the fan. However, no numerical problem occurs if the fan station itself is given as the value of x/C. This is because the program uses the total pressure upstream of the fan to compute the duct inner surface pressure.

In the Card No. 11 NRUN should lie between 0 and 999, NPHI should lie between 0 and 5. If NPHI = 0 then no pressure calculations are performed. The advance ratio J must be greater than zero. For α the only requirement is that $\cos \alpha \neq 0$. ϕ values should lie between $0 \leq \phi \leq 180^{\circ}$ because of symmetry.

3. PART II DESCRIPTION:

Part II of the program determines the pressure and forces on the wing for three cases. They are for the complete wing, the wing with a duct in the absence of the fan and the wing in the presence of the fan. For thelast case the program uses the Part I results for the values of the C_n coefficients of the Glauert series representing the duct bound vorticity, the strength of the vortex cylinder trailing from the duct trailing edge and the fan angle of attack measured from the fan axis of rotation. The choice as to which of the three cases the program should analyse is governed by the values of the two indices NNN and NEXT built into the main program.

Part II is made-up of a main program and six subroutines. The relationship between the main program and the subroutines is given in Fig. 16. The subroutines used and their functions are given below.

MAIN program controls the flow of the program and reads and prints the input data. It nondimensionalises the vortex lattice data with respect to the root chord. The flow of the program is governed by the indeces NNN and NEXT. If NNN is positive then only velocities in the duct due to the wing are calculated. If NNN is zero then only the forces on the wing are calculated. If NNN is negative then both the forces on the wing and the upwash in the duct due to the wing are computed. If the index NEXT is zero or negative then the effect of the

fan on the wing is not calculated. If NEXT is positive then the pressure and forces on the wing in the presence of the fan are calculated.

WING computes the pressure and forces on the various points on the wing and calls other subroutines to get the over all lift and moment coefficients. This subroutine also prints out the output.

MATINV performs matrix inversion and solves simultaneous equations.

VARSIM uses the Simpson's method with unequal intervals for numerical integration. Some statements have been built into the program to tackle the specific example considered.

CURFIT does some curve fitting to avoid the numerical problems due to theoretically predicted infinite pressure at the flat plate leading edge.

EFFECT computes the effect of the fan on the wing. This uses the fan data and some of the results obtained in Part I.

ELLIPS does a table look up for the complete elliptic integrals of the first and second kind given the arguments of the elliptic integrals. This subroutine is the same as the subroutine ELLIPS of Part I.

4. INPUT DESCRIPTION:

This sub-section describes the form in which the input to Part II is to be given. It gives the card number, the variable name in order of punching and the format in which the various wing parameters are to be input.

CARD NO. 1 contains the number of vortices N in the wing vortex lattice, the number of control points MN, on the wing and the number of spanwise stations M. The format for all the three variables is I 3.

CARD No. 2 contains more detailed information regarding the vortex lattice. There are as many cards in this as the number of vortices. Each card contains the x location of the vortex (XV(I)), the distance between the successive control point, (DELTA X (I)), the x location of the control point (X P(I)), the y location of the vortex centre (YV (I)), the span of the vortex (S(I)) and the y location of the control point (YP(I)). The format for all the variables is F 10.6.

CARD NO. 3 is present only when velocity at various control points in the duct in the absence of the fan, is to be determined. It contains the x location of the control point in the duct (X P(I)) followed by the y location (Y P(I)). The format is F 10.6 for both the variables. There are as many cards as the number of control points in the duct at which the velocity is to be determined.

CARD NO. 4 gives the number of vortices plus one (N 1(I)) at each spanwise station. This card contains M values. The format for all the values is I 5.

CARD NO. 5 gives the identification number L 1 (I) of the first vortex in each spanwise station. There are M values in this card and the format in which they are to be punched is I 5.

CARD NO. 6 is present only when the effect of the fan on the wing is to be determined. It contains the six C_n coefficients determined in Part I of the program. The format in which each of them is to be punched is F 10.5.

The data for the wing of Reference |5|, without the duct, is given in Table 2. Table 3 gives the input data for the above wing with the duct. It should be noted that values of the vortex and control point locations are one fifth of the actual values. This was because they were measured from a scaled down diagram. The program converts the value to the actual values and then non-dimensionalises them with respect to the root chord.

In addition to the above input some information has to be input in the body of the program. In the main program, the following have to be given, the value of the wing angle of attack (A1) in degrees, the value of the fan angle of attack (ALF) in radians and the value GAM of the vortex cylinder trailing from the duct trailing edge. The latter two values are to be given only when the effect of the fan on the wing is

to be determined. In subroutine EFFECT the following data has to be given. The x location of the fan (XF), the y location of the fan (YF), both non-dimensionalised with respect to the rest chord, the root chord (CROOT), the C/D of the duct (CD) and the duct trailing edge radius (RTE).

The only limitations on the wing input data are the memory limitations of the computer and size of the matrix MATINV can invert.

5. PROGRAM LISTING:

The following pages give the program listing. The table below will act as a table of contents for the program listing.

```
      SUBROUTINE MAIN(NNN1)
$IBFTC MAIN
C     DUCTED PROPELLER ANALYSIS PROGRAM
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(5)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25)
     1,UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
C
      COMMON MZZZ,CD,RU, R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR2/C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1         RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR4/UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
120   FORMAT(2I5,7F10.6)
997   FORMAT(///10X,42HPROGRAM DID NOT CONVERGE ON INFLOW PROFILE///)
999   FORMAT(///10X,*ERROR - EXCECUTION TERMINATED*///)
C
C     INITIALIZATION OF SUBROUTINES
C
C
1      DUM=0.0
      MZZZ=0
      CALL ELLIPS(DUM,DUM,DUM)
      CALL LAMBDA (DUM,DUM,DUM)
      CALL PKL  (DUM,P)
      CALL CLALF(0)
      MZZZ=1
C
      PI=3.1415926
      RAD=180./PI
C
29    NERR=0
      CALL INPUT
      IF(NERR)28,28,999
C
28    CSALF=COS(ALF/RAD)
      ARJV=ARJ
      ARJVP =ARJP
      ARJ=ARJ*CSALF
      ARJP=ARJP*CSALF
      DO30 K=1,NZ
30    UV(K)=2.0
      NTIME=1
      CORCB=2.0
31    DO 32 K=1,NZ
      BPI=RB(K)/ARJP/UV(K)
      ABPI=ATAN(BPI)
      ALPHA(K)=(ABPI-BTA(K))*RAD
```

```
 BPI=BPI*BPI
BPI=SQRT(BPI+1.0)
J=K
CALL CLALF(J)
IF(NERR-1)25,999,999
CONTINUE
GRV(K)=0.5*CL*BR(K)*UV(K)*BPI
CONTINUE
GV(NZ)=CRV(NZ)*BLD/PI/ARJP+1.
AGV=1.0
IF(GV(NZ))133,233,233
GV(NZ)=-GV(NZ)
AGV=-1.
GV(NZ)=AGV*SQRT(GV(NZ))-1.
AGV=1.
SGV=1.0
DO35J=1,MZ
K=NZ-J
L=K+1
  SGV=SGV+GV(L)
SGV2=SGV*SGV
GRVD=GRV(K)-GRV(L)
GRVD=GRVD*BLD/PI/ARJP
DUM=SGV2+GRVD
AGV=1.0
IF(DUM.LT.0.0)AGV=-1.0
GV(K)=AGV*SQRT(AGV*DUM)-SGV
CONTINUE
DO38 N=1,6
SA(N)=0.
SAS(N)=0.
DO40 N=1,6
DO39M=1,MZ
SA(N)=SA(N)+A(M,N)*GV(M)
SAS(N)=SAS(N)+AS(M,N)*GV(M)
CONTINUE

DO41 N=1,6
SA(N)=SA(N)+D(N)*CORCB
SAS(N)=SAS(N)+DS(N)*CORCB
GAM=GV(NZ)
CALL CNCOEF(GAM,C)
DO 141 N=1,6
SA(N)=SA(N)-D(N)*CORCB
SAS(N)=SAS(N)-DS(N)*CORCB

COMPUTE CORRECTIONS FOR LOW ADVANCE RATIOS          (COR 1)

DUM=XPRES(1)
DUMM=RA(1)
XPRES(1)=XP
RA(1)=1.0
CALL GAMCYL(CD,XP,1,RB,UG,1,XPRES,UGP,PRP,RA)
IRC=-1
CALL VTXRNG(CD,XP,IRC,RB,C,UGD,XPRES,P)
```

```
      CORJ=UGD(1)*GAM+UGP(1,1)*GAM+1.0
      IF(CORJ.LT.1.0)CORJ=1.0
C
C     COMPUTE CORRECTION FOR CENTRE BODY INDUCED VELOCITIES (CORCB)
C
      RA(1)=0.0
      CALL GAMCYL(CD,XP,1,PA,UG,U, XPRES,UGP,RRP,RA)
      IRC=1
      CALL VTXRNG(CD,XP,IRC,RA,C,UGD,XPRES,P)
      CORCB=UGD(1)*GAM+UGP(1,1)*GAM+1.0
       DO 142J=1,MZ
142   CORCB=CORCB+GV(J)/2.0
      IF(CORCB.LT.1.0)CORCB=1.0
      XPRES(1)=DUM
      RA(1)=DUMM
C
      CDP=CD*RRP
      CALL VTXRNG(CDP,XP,NZ,RU,C,UGD,XPRES,P)
98    IRT=0
      SUMG=0.
      DO 42J=1,MZ
42      SUMG=SUMG+GV(J)
      DO 45J=1,MZ
      UNV=1.+(UG(J)*GAM)+(UGD(J)*GAM)+UQD(J)*CORJ+UCB(J)*CORCB+SUMG/2.0
      SUMG=SUMG-GV(J)
      DELV=UV(J)-UNV
      DELV=DELV/UNV
      DELV =ABS(DELV)
      IF(DELV-EPS)45,45,44
44    IRT=IRT+1
45    UV(J)=(UV(J)+UNV)/2.
      J=NZ
      UNV=1.+(UG(J)*GAM)+(UGD(J)*GAM)+UQD(J)*CORJ+UCB(J)*CORCB
      DELV=UV(NZ)-UNV
      DELV=DELV/UNV
      DELV=ABS(DELV)
      UV(NZ)=(UV(NZ)+UNV)/2.
      IF(DELV-EPS)46,46,47
47     IRT=IRT+1
46    IF(IRT)50,60,50
50    NTIME=NTIME+1
       IF(NTIME-51)31,60,60
60    CALL OUTPUT
      IF(NTIME-51)68,69,68
68    NERR=0
      GO TO 31
69    IF(NERR)70,70,996
70     PRINT 600,GAM
600    FORMAT(1X,*  GAM=*,E15.7)                                        DUC01560
       PRINT 601,(C(I),I=1,6)                                           DUC01561
601    FORMAT(1X,*  C(I)*4X,6E15.7)                                     DUC01562
                                                                        DUC01563
```

DUC01564

```
      READ120,NRUN,NPHI,ARJ,ALF,(PHI(J),J=1,5)
      ARJP=ARJ/PI/RRP
      NPAG=0
      IF(NPHI.GT.5)NPHI=5
      IF(NRUN)28,29,28
C
C     ERROR STOP
995   WRITE(6,997)
999   WRITE(6,998)
      STOP
      END
```

```
$IBFTC SU01
      SUBROUTINE INPUT
C
      DIMENSION B(6),BS(6),SA(6) ,SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25),
     1          UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
      DIMENSION RE(4)

      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR2/C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1             RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR4/UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
101   FORMAT(15H1      RUN NUMBER,I5,49X,4HPAGE,I3//)
102   FORMAT(12H          INPUT/5X,65HDUCT GEOMETRY...      C/D        XP/C
     1  T/C       RTE/RP      RCB/RP)
103   FORMAT(10X,20HCAMBER COEFFICIENTS,,4F10.6//)
104   FORMAT(5X,22HPROPELLER GEOMETRY... ,I3,1X,6HBLADES//)
105   FORMAT(25X,34HR/RP          B/RP          BETA      TH/CHD)
106   FORMAT(/5X,47HDEFINITION OF SYMBOLS USED IN TABULAR OUTPUT...//)
107   FORMAT(10X,66HR/RP          RADIAL PROPELLER STATUON IN FRACTION OF PR
     1OPELLER RADIUS)
108   FORMAT(10X,57HB/RP          PROPELLER CHORD IN FRACTION OF PROPDLLER R
     1ADIUS)
109   FORMAT(1,X,36HBETA          PRIPELLER PITCH IN DEGREES)
110   FORMAT(10X,50HTH/CHD        PROPELLER BLADE THICKNESS-TO-CHORD RATIO)
111   FORMAT(/5X,22HCENTERBODY GEOMETRY...2X5HLCB/C,5X5HXCB/C,3X8HRMAX/L
     1CB,2X9HX(RMAX)/C/25X,4F10.5)
112   FORMAT(/5X,*CONVERGENCE CRITERION..EPSILON=*,F10.5)
114   FORMAT(10X1HV9X20HFREE STREAM VELOCITY)
122   FORMAT(10X,31HU             T/TAL INFLOW VELOCITY)
124   FORMAT(10X,48HGAM/V         STRENGTH OF INTERNAL VORTEX CYLINDER N)
125   FORMAT(10X,34HALPHA         ANGLE OF ATTACK,DEGREES )
126   FORMAT(10X,60HDELTA P/Q RISE IN TOTAL PRESSURE ACROSS PROPELLER NO
     1RMALIZED/25X,31HON FREE STREAM DYNAMIC PRESSURE)
127   FORMAT(10X,53HCTP(D)        THRUST COEFFICIENT ON PROPELLER IN THE DUC
     1T)
128   FORMAT(10X,40HCTD(P)        THRUST COEFFECIENT ON THE DUCT)
129   FORMAT(10X34HCTDP          TOTAL THRUST COEFFECIENT)
130   FORMAT(10X,65HCTD(P)'       THRUST COEFFECIENT ON DUCT INCLUDING PRESS
     1URE THRUST ON/25X,29HTHE DUCT AFT OF THE PROPELLER)
131   FORMAT( 10X,*  CTDP'         TOTAL THRUST COEFFECIENT INCLUDING PRESSU
     1RE THRUST*)
132   FORMAT(10X,*CNDP          TOTAL NORMAL FORCE COEFFICIENT*)
133   FORMAT(10X,* CMDP          TOTAL PITCHING MOMENT COEFFUCIENT*)
134   FORMAT(10X,*J             ADVANCE RATIO *)
135   FORMAT(1,X,*J'           RATIO OF V TO PROPELLER TIP SPEED *)
146   FORMAT(20A4)
```

```
149   FORMAT(10X,20A4//)
C
240   FORMAT(F10.3,F8.3,F10.4,F8.3,F8.4,2(2X,1PE12.5))
241   FORMAT(I10,6F10.6)
244   FORMAT(5X,6(1PE13.6))
250   FORMAT(20X,5F10.6//)
251   FORMAT(20X,2F10.6,F10.3,F10.5)
520   FORMAT(2I5,7F10.4)
521   FORMAT(8F10.4)
522   FORMAT(10I5)
C
750   FORMAT(///10X,*NUMBER OF BLADES IN ERROR, NBLD =*,I5)
751   FORMAT(///10X,*CONVERGENCE CRITERION MUST BE GREATER THAN 0.0*)
752   FORMAT(///10X,* ANGLE OF ATTACK MUST BE LESS THAM 90DEGREES*)
753   FORMAT(///10X,*ADVANCE RATIO MUST BE GREATER THAN 0.0*)
C
20    READ(5,148)TALK
      READ(5,521) CD,XR,TC,RRP,RCBRR,EPS
      READ(5,521) R0,R1,R2,R3
      READ(5,521) ELCBC,XCB,RMAX,XR
      READ(5,522)NBLD,NZ,NZP,IP,NPRES,NPRINT
      READ(5,521) (RB(J),J=1,NZP)
      READ(5,521)(BR(J),J=1,NZP)
      READ(5,521)(BTA(J),J=1,NZP)
      READ(5,521)(TCBLD(J),J=1,NZP)
      READ(5,521)(XPRES(N),N=1,IP)
      READ(5,520)NRUN ,NPHI,ARJ,ALF,(PHI(J),J=1,5)
C
C     CHECK INPUT DATA
C
      IF(NPHI.GT.5) NPHI=5
      IF(NBLD) 700,700,701
700   PRINT750,NBLD
      NERR=1
701   IF(EPS)702,702,703
702   PRINT 751
      NERR=1
703   IF(ALF-90.0)704,705,705
705   PRINT752
      NERR=1
704   IF(ARJ)706,706,707
706   PRINT 753
      NERR=1
707   IF(NERR)708,708,999
708   CONTINUE
      NPAG=0
      PRINT101,NRUN,NPAG
      PRINT149,TALK
C
C      INITIALISATION OF SUBROUTINES
C
      MZZZ=0
       CALL HUB(CD,XR,XCB,ELCBC,RMAX,IR,XPRES,RB,RRP,UCB,VC3)
      IF(NERR.GE.1) GO TO 999
      CALL ALFRNG(CD,50,SC,UGA,XPRES)
```

```
 MZZZ=1
DO 40 K=1,6
D(K)= VCB(K)
DS(K)=UCB(K)

CALL PROP(NZP,RB,BR,BTA,TCBLD,RRP,RCBRP,NZ,RA)

 COMPUTE THE CONSTANT PORTION OF THE INFLOW PROFILE

CDP=CD*RRP
CALL GAMCYL(CDP,XP,NZ,RP,UG,0,XPRES, UGP,RRP,RA)
CALL SRCRNG(CD,XP, TC,NZ,RB,UOD)
CALL HUB(CD,XP,XCB,ELCBC,RMAX,NZ,XPRES,RB,RRP,UCB,VCB)

 COMPUTE THE FOURIER COEFFICIENTS DUE T/ THE TRAILING VORTICITY

CRBN=CD*2.
CALL BNCOEF (CRBN,B)
NOUT=6
 N=50
NCYL=NZ-1
CALL ANCOEF(NCYL,N,RA,XP,CD,RRP,NRUN,0,BS,A,AS)
DO 30 K=1,NZ
BR(K)=BR(K)/RRP
RB(K)=RB(K)/RRP
BTA(K)=90.0-BTA(K)
BTA(K)=BTA(K)/RAD
RETURN
END
```

```
$IBFTC SU02
      SUBROUTINE PKL (CD,P)
C
      DIMENSION  P(6,6),A(6,6,7),C(7)
C
      COMMON MZZZ
C
      IF (MZZZ) 1,2,1
2     DO 10K=1,6
      DO 10 L=1,6
      DO10 M=1,7
10    A(K,L,M)=^.0
C
      A(1,1,2)=.02683
      A(1,3,2)=.01343
      A(1,5,2)=-.00001
      A(2,1,2)= .05366
      A(2,2,2)= .02987
      A(2,4,2)=-.00304
      A(3,1,2)= .006^8
      A(3,3,2)= .004^3
      A(3,5,2)=^.00099
      A(4,1,2)=^.000^5
      A(4,2,2)=^.00101
      A(4,4,2)= .00148
      A(4,6,2)=-.00049
      A(5,1,2)=-.00^01
      A(5,3,2)=^.00049
      A(5,5,2)= .00079
      A(6,4,2)=-.00029
      A(6,6,2)= .00049
C
      A(1,1,3)= .07281
      A(1,3,3)= .03644
      A(1,5,3)=-.00^04
      A(2,1,3)=.14561
      A(2,2,3)= .08526
      A(2,4,3)=^.01252
      A(2,6,3)= .00006
      A(3,1,3)=.02491
      A(3,3,3)= .01655
      A(3,5,3)=-.00411
      A(4,1,3)=-.00016
      A(4,2,3)=^.00417
      A(4,4,3)= .006^9
      A(4,6,3)=^.002^0
      A(5,1,3)=^.00013
      A(5,3,3)=-.00206
      A(5,5,3)= .00318
      A(6,2,3)= .00002
      A(6,4,3)=-.00120
      A(6,6,3)=.00197
C
      A(1,1,4)= .11925
      A(1,3,4)=.05945
      A(1,5,4)= .00019
      A( 2,1,4)= .23849
```

A(2,2,4)= .14668
A(2,4,4)=^.02765
A(2,6,4)= .00021
A(3,1,4)= .05488
A(3,3,4)= .037^2
A(3,5,4)=-.00965
A(4,1,4)= .00071
A(4,2,4)=^.00922
A(4,4,4)= .01418
A(4,6,4)=^.00463
A(5,1,4)=^.00042
A(5,3,4)=-.00482
A(5,5,4)= .00732
A(6,1,4)=-.00005
A(6,2,4)= .00005
A(6,4,4)=-.00278
A(6,6,4)=.00450

A(1,1,5)=.16016
A(1,3,5)= .07907
A(1,5,5)= .00107
A(2,1,5)= .32032
A(2,2,5)= .206^9
A(2,4,5)=-.04629
A(2,6,5)= .00035
A(3,1,5)= .09186
A(3,3,5)= .06338
A(3,5,5)=-0.01766
A(4,1,5)= .004^2
A(4,2,5)=-.01544
A(4,4,5)= .02592
A(4,6,5)=^.00855
A(5,1,5)=^.00072
A(5,3,5)=-.00883
A(5,5,5)= .01338
A(6,1,5)=^.00025
A(6,2,5)=.00008
A(6,4,5)=-.00512
A(6,6,5)= .00814

A(1,1,6)= .22278
A(1,3,6)= .10634
A(1,5,6)= .00536
A(2,1,6)= .44556
A(2,2,6)= .30830
A(2,4,6)=^.08513
A(2,6,6)=^.00059
A(3,1,6)= .17105
A(3,3,6)= .12346
A(3,5,6)=^.03845
A(4,1,6)= .02020
A(4,2,6)=^.02784
A(4,4,6)= .05692
A(4,6,6)=-.01932
A(5,1,6)=.00079
A(5,3,6)=^.01858
A(5,5,6)= .02935
A(6,1,6)=^.00123
A(6,2,6)=^.00011

8

```
C
      A(1,1,7)= .26624
      A(1,3,7)= .12146
      A(1,5,7)= .01214
      A(2,1,7)= .53248
      A(2,2,7)=.38891
      A(2,4,7)=-.11898
      A(2,6,7)=-.00429
      A(3,1,7)= .24533
      A(3,3,7)= .18468
      A(3,5,7)=-.06210
      A(4,1,7)= .04662
      A(4,2,7)=-.03871
      A(4,4,7)= .09563
      A(4,6,7)=-.03416
      A(5,1,7)= .00738
      A(5,3,7)=-.02992
      A(5,5,7)= .05270
      A(6,1,7)=-.00194
      A(6,2,7)=-.00088
      A(6,4,7)=-.01918
      A(6,6,7)= .02998
C
      C(1)=0.0
       C(2)=0.25
       C(3)=0.50
      C(4)=0.75
      C(5)=1.00
      C(6)=1.50
       C(7)=2.00
      GOTO 71
C
1      DO 20 J=1,7
      IF (C(J)-CD) 20,19,21
20    CONTINUE
      GO TO 21
19    M=J
      GOTO 100
21    M=J-1
      N=J
      DELT =C(N)-C(M)
      DIFF =CD-C(M)
      DELTA=DIFF/DELT
      DO 30K=1,6
      DO 30 L=1,6
      P(K,L)=A(K,L,M)+(DELTA*(A(K,L,N)-A(K,L,M)))
30    CONTINUE
      GO TO 71
100   DO 40 K=1,6
      DO 40L=1,6
      P(K,L)=A(K,L,M)
40    CONTINUE
71    RETURN
      END
```

8

```
C
      A(1,1,7)= .26624
      A(1,3,7)= .12146
      A(1,5,7)= .01214
      A(2,1,7)= .53248
      A(2,2,7)=.38891
      A(2,4,7)=-.11898
      A(2,6,7)=-.00429
      A(3,1,7)= .24533
      A(3,3,7)= .18468
      A(3,5,7)=-.06210
      A(4,1,7)= .04662
      A(4,2,7)=^.03871
      A(4,4,7)= .09563
      A(4,6,7)=^.03416
      A(5,1,7)= .00738
      A(5,3,7)=^.02992
      A(5,5,7)= .05270
      A(6,1,7)=^.00194
      A(6,2,7)=^.00088
      A(6,4,7)=-.01918
      A(6,6,7)= .02998
C
      C(1)=0.0
       C(2)=0.25
       C(3)=0.5^
      C(4)=0.75
      C(5)=1.0^
      C(6)=1.50
       C(7)=2.0^
      GOTO 71
C
1      DO 20 J=1,7
      IF (C(J)-CD) 20,19,21
20    CONTINUE
      GO TO 21
19    M=J
      GOTO 100
21    M=J-1
      N=J
      DELT =C(N)-C(M)
      DIFF =CD-C(M)
      DELTA=DIFF/DELT
      DO 30K=1,6
      DO 30 L=1,6
      P(K,L)=A(K,L,M)+(DELTA*(A(K,L,N)-A(K,L,M)))
30    CONTINUE
      GO TO 71
100   DO 40 K=1,6
      DO 40L=1,6
      P(K,L)=A(K,L,M)
40    CONTINUE
71    RETURN
      END
```

```
$IBFTC 03
      SUBROUTINE ELLIPS (AKSQ,TK,TE)
C
C     SUB. ELLIPS --TABLE LOOK UP OG ELLIPTIC INTEGERALS
C
      DIMENSION CKK(100),CK(100),CE(100)
      COMMON MZZZ
      IF (MZZZ) 73,10,3
10    CONTINUE
C
C          CKK= ARGUMENT OF ELLIPTIC INTEGRALS
C
      CKK(1)=0.00
      CKK(2)=0.01
      CKK(3)=0.02
      CKK(4)=0.03
      CKK(5)=0.04
      CKK(6)=0.05
      CKK(7)=0.06
      CKK(8)=0.07
      CKK(9)=0.08
      CKK(10)=0.09
      CKK(11)=0.10
      CKK(12)=0.11
      CKK(13)=0.12
      CKK(14)=0.13
      CKK(15)=0.14
      CKK(16)=0.15
      CKK(17)=0.16
      CKK(18)=0.17
      CKK(19)=0.18
      CKK(20)=0.19
      CKK(21)=0.20
      CKK(22)=0.21
      CKK(23)=0.22
      CKK(24)=0.23
      CKK(25)=0.24
      CKK(26)=0.25
      CKK(27)=0.26
      CKK(28)=0.27
      CKK(29)=0.28
      CKK(30)=0.29
      CKK(31)=0.30
      CKK(32)=0.31
      CKK(33)=0.32
      CKK(34)=0.33
      CKK(35)=0.34
      CKK(36)=0.35
      CKK(37)=0.36
      CKK(38)=0.37
      CKK(39)=0.38
       CKK(40)=0.39
      CKK(41)=0.40
      CKK(42)=0.41
      CKK(43)=0.42
```

```
CKK(44)=0.43
CKK(45)=0.44
CKK(46)=0.45
CKK(47)=0.46
CKK(48)=0.47
CKK(49)=0.48
CKK(50)=0.49
CKK(51)=0.50
CKK(52)=0.51
CKK(53)=0.52
CKK(54)=0.53
CKK(55)=0.54
CKK(56)=0.55
CKK(57)=0.56
CKK(58)=0.57
CKK(59)=0.58
CKK(60)=0.59
CKK(61)=0.60
CKK(62)=0.61
CKK(63)=0.62
CKK(64)=0.63
CKK(65)=0.64
CKK(66)=0.65
CKK(67)=0.66
CKK(68)=0.67
CKK(69)=0.68
CKK(70)=0.69
CKK(71)=0.70
CKK(72)=0.71
CKK(73)=0.72
CKK(74)=0.73
CKK(75)=0.74
CKK(76)=0.75
CKK(77)=0.76
CKK(78)=0.77
CKK(79)=0.78
CKK(80)=0.79
CKK(81)=0.80
CKK(82)=0.81
CKK(83)=0.82
CKK(84)=0.83
CKK(85)=0.84
CKK(86)=0.85
CKK(87)=0.86
CKK(88)=0.87
CKK(89)=0.88
CKK(90)=0.89
CKK(91)=0.90
CKK(92)=0.91
CKK(93)=0.92
CKK(94)=0.93
CKK(95)=0.94
CKK(96)=0.95
CKK(97)=0.96
CKK(98)=0.97
```

```
      CKK(99)=0.98
      CKK(100)=0.99
C
C     CK=COMPLETE ELLIPTIC INTEGRAL OF FIRST KIND
      CK(1)=1.570796
      CK(2)=1.574746
      CK(3)=1.578740
      CK(4)=1.582780
      CK (5)=1.586868
      CK(6)=1.591003
      CK(7)=1.595188
      CK(8)=1.599423
      CK(9)=1.603710
      CK(10)=1.608049
      CK(11)=1.612441
      CK(12)=1.616889
      CK(13)=1.621393
      CK(14)=1.625955
      CK(15)=1.630576
      CK(16)=1.635257
      CK(17)=1.640000
      CK(18)=1.644806
       CK(19)=1.649678
      CK(20)=1.654617
      CK(21)=1.659624
      CK(22)=1.664701
      CK(23)=1.669850
      CK(24)=1.675073
      CK(25)=1.680373
      CK(26)=1.685750
      CK(27)=1.691208
      CK(28)=1.696749
      CK(29)=1.702374
      CK(30)=1.708087
      CK(31)=1.713889
      CK(32)=1.719785
      CK(33)=1.725776
      CK(34)=1.731865
      CK(35)=1.738055
      CK(36)=1.744351
      CK(37)=1.750754
      CK(38)=1.757269
      CK(39)=1.763898
      CK(40)=1.770647
      CK(41)=1.777519
      CK(42)=1.784519
      CK(43)=1.791650
      CK(44)=1.798918
      CK(45)=1.806328
      CK(46)=1.813884
       CK(47)=1.821593
      CK(48)=1.829460
      CK(49)=1.837491
       CK(50)=1.845694
      CK(51)=1.854075
```

CK(52)=1.862641
CK(53)=1.871400
CK(54)=1.880361
CK(55)=1.889533
CK(56)=1.898925
CK(57)=1.908547
CK(58)=1.918410
CK(59)=1.928526
CK(60)=1.938908
CK(61)=1.949568
CK(62)=1.960521
CK(63)=1.971783
CK(64)=1.983371
CK(65)=1.995303
CK(66)=2.007598
CK(67)=2.020279
CK(68)=2.033369
CK(69)=2.046894
CK(70)=2.060882
CK(71)=2.075363
CK(72)=2.090373
CK(73)=2.105948
CK(74)=2.122132
CK(75)=2.138970
CK(76)=2.156516
CK(77)=2.174827
CK(78)=2.193971
CK(79)=2.214022
CK(80)=2.235068
CK(81)=2.257205
CK(82)=2.280549
CK(83)=2.305232
CK(84)=2.331409
CK(85)=2.359264
CK(86)=2.389016
CK(87)=2.420933
CK(88)=2.455338
CK(89)=2.492635
CK(90)=2.533335
CK(91)=2.578092
CK(92)=2.627773
CK(93)=2.683551
CK(94)=2.747073
CK(95)=2.820752
CK(96)=2.908337
CK(97)=3.016112
CK(98)=3.155875
CK(99)=3.354141
CK(100)=3.695637

CE=COMPLETE ELLIPTIC INTEGERAL OF SECOND KIND

CE(1)=1.570796
CE(2)=1.566862
CE(3)=1.562913

CE(4)=1.558948
CE(5)=1.554969
CE(6)=1.550973
CE(7)=1.546963
CE(8)=1.542936
CE(9)=1.538893
CE(10)=1.534833
CE(11)=1.530758
CE(12)=1.526665
CE(13)=1.5225550
CE(14)=1.518428
CE(15)=1.514284
CE(16)=1.510122
CE(17)=1.505942
CE(18)=1.501743
CE(19)=1.497526
CE(20)=1.493290
CE(21)=1.489035
CE(22)=1.4847610
CE(23)=1.480466
CE(24)=1.476152
CE(25)=1.471818
CE(26)=1.467462
CE(27)=1.463086
CE(28)=1.458688
CE(29)=1.454269
CE(30)=1.449827
CE(31)=1.445363
CE(32)=1.440876
CE(33)=1.436366
CE(34)=1.431832
CE(35)=1.477274
CE(36)=1.422691
CE(37)=1.418083
CE(38)=1.413450
CE(39)=1.408791
CE(40)=1.404105
CE(41)=1.399392
CE(42)=1.394652
CE(43)=1.389883
CE(44)=1.385086
CE(45)=1.380259
CE(46)=1.375402
CE(47)=1.370515
CE(48)=1.365596
CE(49)=1.360645
CE(50)=1.355661
CE(51)=1.350644
CE(52)=1.345592
CE(53)=1.340505
CE(54)=1.335382
CE(55)=1.330223
CE(56)=1.325024
CE(57)=1.319788
CE(58)=1.314511

```
CE(59)=1.309192
CE(60)=1.303832
CE(61)=1.298428
CE(62)=1.292979
CE(63)=1.287484
CE(64)=1.281942
CE(65)=1.276350
CE(66)=1.270707
CE(67)=1.265013
CE(68)=1.259263
CE(69)=1.253458
CE(70)=1.247595
CE(71)=1.241671
CE(72)=1.235684
CE(73)=1.229632
CE(74)=1.223512
CE(75)=1.217321
CE(76)=1.211056
CE(77)=1.204714
 CE(78)=1.198290
CE(79)=1.191781
CE(80)=1.185183
CE(81)=1.178490
CE(82)=1.171697
CE(83)=1.164798
CE(84)=1.157787
CE(85)=1.150656
CE(86)=1.143396
CE(87)=1.135998
CE(88)=1.128451
CE(89)=1.1SS742
CE(90)=1.112856
CE(91)=1.104775
 CE(92)=1.096478
CE(93)=1.087938
CE(94)=1.079121
CE(95)=1.069986
CE(96)=1.060474
CE(97)=1.050502
CE(98)=1.039947
CE(99)=1.028595
CE(100)=1.015994

GO TO 30
IF (AKSQ-.99)20,20,21
PARA=0.25*(1.0-AKSQ)
TEST =1.00E-07
IF(PARA-TEST)701,702,702
PARA =TEST
ZLP=ALOG(4./PARA )
TK =ZLP*0.5*(1.+PARA)-PARA
TE=1.0+(ZLP*PARA)-PARA
GO TO 30
JA=100.0*AKSQ
JA=1+JA
```

```
      IF(CKK(JA)-AKSQ)22,23,22
23    TK=CK(JA)
      TE=CE(JA)
      GO TO 30
22    CON=(AKSQ-CKK(JA))/(CKK(JA+1)-CKK(JA))
      TK=CK(JA)+CON*(CK(JA+1)-CK(JA))
      TE=CE(JA)+CON*(CE(JA+1)-CE(JA))
      GO TO 30
73    IF(AKSQ-.1)721,721,720
721   PARA=.25*ASSQ
      GO TO 700
720   AKSQ=1.-AKZZ
      GO TO20
30    CONTINUE
      RETURN
      END
```

```
$IBFTC 04
      SUBROUTINE LAMBDA (XP,ZP,YP)
C
C TABLE LOOKUP OF HEUMAN LAMBDA FUNCTION
C
      DIMENSION Y(19,19),X(19),Z(19)
      COMMON MZZZ
      IF (MZZZ )20,10,20
10     CONTINUE
C
C        X=ARC SIN  K(DEGREES)
C
      X(1)=0.000000
      X(2)=5.000000
      X(3)=10.000000
      X(4)=15.000000
      X(5)=20.000000
      X(6)=25.000000
      X(7)=30.000000
      X(8)=35.000000
      X(9)=40.000000
      X(10)=45.000000
      X(11)=50.000000
      X(12)=55.000000
      X(13)=60.000000
      X(14)=65.000000
      X(15)=70.000000
      X(16)=75.000000
      X(17)=80.000000
      X(18)=85.000000
      X(19)=90.000000

         Z=BETA (DEGREES)

      Z(1)=0.000000
      Z(2)=5.000000
      Z(3)=10.000000
      Z(4)=15.000000
       Z(5)=20.000000
      Z(6)=25.000000
      Z(7)=30.000000
      Z(8)=35.000000
      Z(9)=40.000000
      Z(10)=45.000000
      Z(11)=50.000000
      Z(12)=55.000000
      Z(13)=60.00000
      Z(14)=65.000000
      Z(15)=70.000000
      Z(16)=75.000000
      Z(17)=80.000000
      Z(18)=85.000000
      Z(19)=90.000000

       Y=HEUMAN LAMBDA FUNCTION
```

```
Y(1,1)=0.000000
Y(1,2)= 0.087156
Y(1,3)=0.173648
Y(1,4)=0.258819
Y(1,5)=.342020
Y(1,6)=.422618
Y(1,7)=.5
Y(1,8)=.573576
Y(1,9)=.642788
Y(1,10)=.707107
Y(1,11)=.766044
Y(1,12)=.819152
Y(1,13)=.866025
Y(1,14)=.906308
Y(1,15)=.939693
Y(1,16)=.965926
Y(1,17)=.984808
Y(1,18)=.996195
Y(1,19)=1.
Y(2,1)=.0
Y(2,2)=.086990
Y(2,3)=.173318
Y(2,4)=.258327
Y(2,5)=.341370
Y(2,6)=.421815
Y(2,7)=.499050
Y(2,8)=.572487
Y(2,9)=.641567
Y(2,10)=.705765
Y(2,11)=.764592
Y(2,12)=.8176
Y(2,13)=.864388
Y(2,14)=.904599
Y(2,15)=.937930
Y(2,16)=.964135
Y(2,17)=.983037
Y(2,18)=.994624
Y(2,19)=1.
Y(3,1)=0.
Y(3,2)= .086495
Y(3,3)=.172332
Y(3,4)=.256858
Y(3,5)=0.339430
Y(3,6)=0.419419
Y(3,7)=0.496219
Y(3,8)=0.569244
Y(3,9)=0.637940
Y(3,10)=0.701786
Y(3,11)=0.760298
Y(3,12)=0.813034
Y(3,13)=0.859602
Y(3,14)=0.899660
Y(3,15)=0.932934
Y(3,16)=0.959244
Y(3,17)=0.978597
```

```
Y(3,18)=0.991511
Y(3,19)=1.000000
Y(4,1)=0.000000
Y(4,2)=0.085677
Y(4,3)=0.170704
Y(4,4)=0.254434
Y(4,5)=0.336231
Y(4,6)=0.415475
Y(4,7)=0.491565
Y(4,8)=0.563926
Y(4,9)=0.632010
Y(4,10)=0.695307
Y(4,11)=0.753346
Y(4,12)=0.805703
Y(4,13)=0.852010
Y(4,14)=0.891969
Y(4,15)=0.925384
Y(4,16)=0.952226
Y(4,17)=0.972787
Y(4,18)=0.988015
Y(4,19)=1.000000
Y(5,1)=0.000000
Y(5,2)=0.084549
Y(5,3)=0.168458
Y(5,4)=0.251092
Y(5,5)=0.331827
Y(5,6)=0.410054
Y(5,7)=0.485184
Y(5,8)=0.556657
Y(5,9)=0.623939
Y(5,10)=0.686540
Y(5,11)=0.744012
Y(5,12)=0.795963
Y(5,13)=0.842073
Y(5,14)=0.882119
Y(5,15)=0.916018
Y(5,16)=0.943918
Y(5,17)=0.966343
Y(5,18)=0.984410
Y(5,19)=1.000000
Y(6,1)=0.000000
Y(6,2)=0.083124
Y(6,3)=0.165625
Y(6,4)=0.246882
Y(6,5)=0.326288
Y(6,6)=0.403252
Y(6,7)=0.477203
Y(6,8)=0.547600
Y(6,9)= 0.613936
Y(6,10)=0.675748
Y(6,11)=0.732623
Y(6,12)=0.784220
Y(6,13)=0.830282
Y(6,14)=0.870676
Y(6,15)=0.905441
```

Y(6,16)=0.934867
Y(6,17)=0.959607
Y(6,18)=0.980779
Y(6,19)=1.000000
Y(7,1)=0.000000
Y(7,2)=0.081425
Y(7,3)=0.162247
Y(7,4)=0.241870
Y(7,5)=0.319707
Y(7,6)=0.395191
Y(7,7)=0.467777
Y(7,8)=0.536953
Y(7,9)=0.602244
Y(7,10)=0.663225
Y(7,11)=0.719533
Y(7,12)=0.770883
Y(7,13)=0.817093
Y(7,14)=0.858117
Y(7,15)=0.894095
Y(7,16)=0.925409
Y(7,17)=0.952751
Y(7,18)=0.977159
Y(7,19)=1.000000
Y(8,1)=0.000000
Y(8,2)=0.079476
Y(8,3)=0.158377
Y(8,4)=0.236134
Y(8,5)=0.312192
Y(8,6)=0.386013
Y(8,7)=0.457086
Y(8,8)=0.524935
Y(8,9)=0.589127
Y(8,10)=0.649283
Y(8,11)=0.705094
Y(8,12)=0.756337
Y(8,13)=0.802903
Y(8,14)=0.844820
Y(8,15)=0.882297
Y(8,16)=0.915757
Y(8,17)=0.945873
Y(8,18)=0.973573
Y(8,19)=1.000000
Y(9,1)=0.000000
Y(9,2)=0.077307
Y(9,3)=0.154073
Y(9,4)=0.229767
Y(9,5)=0.303869
Y(9,6)=0.375880
Y(9,7)=0.445330
Y(9,8)=0.511786
Y(9,9)=0.574862
Y(9,10)=0.634231
Y(9,11)=0.689642
Y(9,12)=0.740932
Y(9,13)=0.788051

```
Y(9,14)=0.831085
Y(9,15)=0.870277
Y(9,16)=0.906056
Y(9,17)=0.939042
Y(9,18)=0.970039
Y(9,19)=1.000000
Y(10,1)=0.000000
Y(10,2)=0.074953
Y(10,3)=0.149408
Y(10,4)=0.222878
Y(10,5)=0.294884
Y(10,6)=0.364976
Y(10,7)=0.432729
Y(10,8)=0.497760
Y(10,9)=0.559735
Y(10,10)=0.618381
Y(10,11)=0.673501
Y(10,12)=0.724985
Y(10,13)=0.772830
Y(10,14)=0.817155
Y(10,15)=0.858217
Y(10,16)=0.896419
Y(10,17)=0.932311
Y(10,18)=0.966576
Y(10,19)=1.000000
Y(11,1)=0.000000
Y(11,2)=0.072455
Y(11,3)=0.144464
Y(11,4)=0.215587
Y(11,5)=0.285399
Y(11,6)=0.353500
Y(11,7)=0.419519
Y(11,8)=0.483125
Y(11,9)=0.544038
Y(11,10)=0.602038
Y(11,11)=0.656976
Y(11,12)=0.708785
Y(11,13)=0.757496
Y(11,14)=0.803241
Y(11,15)=0.846269
Y(11,16)=0.886942
Y(11,17)=0.925731
Y(11,18)=0.963204
Y(11,19)=1.000000
Y(12,1)=0.000000
Y(12,2)=0.069861
Y(12,3)=0.139334
Y(12,4)=0.208034
Y(12,5)=0.275597
Y(12,6)=0.341676
Y(12,7)=0.405958
Y(12,8)=0.468167
Y(12,9)=0.528076
Y(12,10)=0.585512
Y(12,11)=0.640369
```

```
Y(12,12)=0.692612
Y(12,13)=0.742291
Y(12,14)=0.789537
Y(12,15)=0.834576
Y(12,16)=0.877717
Y(12,17)=0.919353
Y(12,18)=0.959944
Y(12,19)=1.000000
Y(13,1)=0.000000
Y(13,2)=0.067226
Y(13,3)=0.134126
Y(13,4)=0.200380
Y(13,5)=0.265684
Y(13,6)=0.329751
Y(13,7)=0.392328
Y(13,8)=0.453192
Y(13,9)=0.512167
Y(13,10)=0.569122
Y(13,11)=0.623985
Y(13,12)=0.676745
Y(13,13)=0.727455
Y(13,14)=0.776237
Y(13,15)=0.823283
Y(13,16)=0.868846
Y(13,17)=0.913240
Y(13,18)=0.956826
Y(13,19)=1.000000
Y(14,1)=0.000000
Y(14,2)=0.064614
Y(14,3)=0.128968
Y(14,4)=0.192809
Y(14,5)=0.255897
Y(14,6)=0.318009
Y(14,7)=0.378946
Y(14,8)=0.438541
Y(14,9)=0.496661
Y(14,10)=0.553214
Y(14,11)=0.608153
Y(14,12)=0.661480
Y(14,13)=0.713246
Y(14,14)=0.763552
Y(14,15)=0.812552
Y(14,16)=0.860443
Y(14,17)=0.907464
Y(14,18)=0.953885
Y(14,19)=1.000000
Y(15,1)=0.000000
Y(15,2)=0.062100
Y(15,3)=0.124009
Y(15,4)=0.185540
Y(15,5)=0.246517
Y(15,6)=0.306778
Y(15,7)=0.366180
Y(15,8)=0.424604
Y(15,9)=0.481959
```

```
Y(15,10)=0.538183
Y(15,11)=0.593247
Y(15,12)=0.647159
Y(15,13)=0.699961
Y(15,14)=0.751731
Y(15,15)=0.802581
Y(15,16)=0.852654
Y(15,17)=0.902119
Y(15,18)=0.951166
Y(15,19)=1.000000
Y(16,1)=0.000000
Y(16,2)=0.059779
Y(16,3)=0.119433
Y(16,4)=0.178329
Y(16,5)=0.237883
Y(16,6)=0.296459
Y(16,7)=0.354475
Y(16,8)=0.411857
Y(16,9)=0.468546
Y(16,10)=0.524506
Y(16,11)=0.579721
Y(16,12)=0.634200
Y(16,13)=0.687972
Y(16,14)=0.741089
Y(16,15)=0.793624
Y(16,16)=0.845669
Y(16,17)=0.897332
Y(16,18)=0.948733
Y(16,19)=1.000000
Y(17,1)=0.000000
Y(17,2)=0.057773
Y(17,3)=0.115479
Y(17,4)=0.173054
Y(17,5)=0.230436
Y(17,6)=0.287571
Y(17,7)=0.344410
Y(17,8)=0.400915
Y(17,9)=0.457055
Y(17,10)=0.512813
Y(17,11)=0.568181
Y(17,12)=0.623166
Y(17,13)=0.677782
Y(17,14)=0.732059
Y(17,15)=0.786036
Y(17,16)=0.839759
Y(17,17)=0.893286
Y(17,18)=0.946677
Y(17,19)=1.000000
Y(18,1)=0.000000
Y(18,2)=0.056256
Y(18,3)=0.112490
Y(18,4)=0.168682
Y(18,5)=0.224814
Y(18,6)=0.280867
Y(18,7)=0.336826
```

```
Y(18,8)=0.392679
Y(18,9)=0.448417
Y(18,10)=0.504034
Y(18,11)=0.559529
Y(18,12)=0.614903
Y(18,13)=0.670162
Y(18,14)=0.725315
Y(18,15)=0.780373
Y(18,16)=0.835352
Y(18,17)=0.890270
Y(18,18)=0.945145
Y(18,19)=1.000000
Y(19,1)=0.000000
Y(19,2)=0.055556
Y(19,3)=0.111111
Y(19,4)=0.166667
Y(19,5)=0.222222
Y(19,6)=0.277778
Y(19,7)=0.333333
Y(19,8)=0.388889
Y(19,9)=0.444444
Y(19,10)=0.500000
Y(19,11)=0.555556
Y(19,12)=0.611111
Y(19,13)=0.666667
Y(19,14)=0.722222
Y(19,15)=0.777778
Y(19,16)=0.833333
Y(19,17)=0.888889
Y(19,18)=0.944444
Y(19,19)=1.000000

GO TO 90
DX=5.0
DZ =5.0
FNX =XP/DX
N=FNX
N=N+1
FNZ=ZP/DZ
M=FNZ
M=M+1
C1=(ZP-Z(M))/DZ
C2=(XP- X(N))/DX
A=(Y(N,M+1)-Y(N,M))*C1
B=(Y(N+1,M)-Y(N,M))*C2
D=C1*C2*(Y(N+1,M+1)-Y(N+1,M)+Y(N,M)-Y(N,M-1))
YP=Y(N,M)+A+B+D
RETURN
END
```

```
$IBFTC 05
      SUBROUTINE HUB (CD,XR,XCB,ELCBC,H,IR,X,RB,RRP,U,V)

      CALCULATION OF VELOCITIES INDUCED BY THE CENTRE BODY
     ** CENTRE BODY IS REPRESENTED AS RANKINE BODY


      DIMENSION X(25), RB(25), U(25),V(25),FU(50), FV(50),D(6),DS(6)
      COMMON MZZZ

  0   FORMAT(///10X,*INPUT CENTREBODY DIMENSION IM  ERROR*///)
  1   FORMAT(///10X,*SUBROUTINE HUB UNABLE TO COMPUTE CENTREBODY GOMETR
     1Y*///)

      IF(MZZZ)3,1,2
      XIZ=(XR-XCB)/ELCBC

     COMPUTE LOCATION OF POINT  SOURCE

      IF(XIZ-H)10,10,11
      NERR=1

         ERROR MESSAGE
      PRINT 700
      RETURN
  1    H2=H*H
      XIZ2=XIZ*XIZ
      A=XIZ-H
      DO 20 J=1,20
      AA=XIZ*H2*SQRT(A*A+H2)
      AA= XIZ2-SQRT(AA)
      AA=SQRT(AA)
       DEL =ABS(AA-A)/AA
      IF(DEL-.001)30,30,19
  )     A=(AA+A)/2.
  )    CONTINUE

        ERROR NESSAGE

      PRINT 701
      NERR=1
      RETURN
       A=AA
       EMV=((XIZ2-A*A)**2)/(4.*XIZ*A)
      RETURN
       IF(IR)100,100,200

        COMPUTE VELOCITIES INDUCED AT THE DUCT REFRENCE CYLINDER
      W=0.5/CD/ELCBC
      NX =-IR
      DO 40 J=1,NX
      XI=X(J)-XCB-XIZ*ELCBC
      XI =XI/ELCBC
       XPA =XI+A
      XMA=XI-A
      RXPA=SQRT(XPA*XPA+W*W)
       RXMA=SQRT(XMA*XMA+W*W)
      RXPA3=RXPA**3
```

```
RXMA3=RXMA**3
U(J)=EMV*(XPA/RXPA3-XMA/RXMA3)
V(J)=EMV*W*(1./RXPA3-1./RXMA3)
CONTINUE
RETURN

  COMPUTE INFLOW VELOCITIES

XI=XR- XCB-XIZ*ELCBC
XI=XI/ELCBC
DO 60 J=1,IR
W=RB(J)*.5/CD/ELCBC/RRP
XPA=XI+A
XMA =XI-A
RXPA=SQRT(XPA*XPA+W*W)
RXMA=SQRT(XMA*XMA+W*W)
RXPA3=RXPA**3
 RXMA3=RXMA**3
U(J)=EMV*(XPA/RXPA3-XMA/RXMA3)
V(J)=EMV*W*(1./RXPA3-1./RXMA3)
PSI=W*W/2.-EMV*(XPA/RXPA-XMA/RXMA )
CONTINUE
RETURN

  COMPUTE D(N) AND D-STAR(N) FOURIER COEFFICIENTD

 W=0.5/CD/ELCBC
N=IR
CN =N
PI =3.1415926
DTH=PI/(CN-1.)
TH =-DTH
DO 80 J=1,N
TH =TH +DTH
CSTH =COS(TH)
XG=0.5*(1.-CSTH)
XI=XG-XCB-XIZ*ELCBC
XI =XI/ELCBC
XPA =XI+A
XMA =XI-A
RXPA =SQRT(XPA*XPA+W*W)
RXMA=SQRT(XMA*XMA+W*W)
RXPA3=RXPA**3
RXMA3=RXMA**3
FU(J)= EMV *( XPA/RXPA3-XMA/RXMA3)
FV(J)=EMV*W*(1./RXPA3-1./RXMA3)
CONTINUE
NOUT =6
CALL FOURCS(FU,FU,N,NOUT,0)
CALL FOURC((FV,FV,N,NOUT,0)
DO 81 J=1,NOUT
D(J)=FV(J)
DS(J)=FU(J)
U(J)=DS(J)
 V(J)=D(J)
RETURN
END
```

```
IBFTC 06
      SUBROUTINE CAMBER(CD,TC,RP)

        SUBROUTINE TO COMPUTE INDUCED CAMBER COEFFICIENTS

      DIMENSION CDRAT(5),DCD(4,5),RP(4)

      DATA CDRAT/0.0,0.25,0.50,0.75,1.0/
      DATADCD(1,1),DCD(1,2),DCD(1,3),DCD(1,4),DCD(1,5)/0.0,0.00149,
     10.0316,0.00505,0.00696/
      DATA DCD(2,1),DCD(2,2),DCD(2,3),DCD(2,4),DCD(2,5) /0.0,0.10897,
     10.22236,0.33414,0.43940/
      DATA DCD(3,1),DCD(3,2),DCD(3,3),DCD(3,4),DCD(3,5) /0.0,0.03558,
     10.07223,0.11022,0.14928/
      DATA DCD(4,1),DCD(4,2),DCD(4,3),DCD(4,4),DCD(4,5) /0.0,-0.00203 ,
     1-0.00539,-0.00948,-0.01375/

      DO 2 J=1,5
      N=J
      IF(CDRAT(J)-CD)2,3,4
      CONTINUE
    4 M=N-1
      DEL=CDRAT(N)-CDRAT(M)
      DIF=CD-CDRAT(M)
      DELTA=DIF/DEL
      DO 10 K=1,4
      RP(K)=DCD(S,M)+DELTA*(DCD(K,N)-DCD(K,M))
    0 CONTINUE
      GO TO 20
    3 M=N
      DO 6K=1,4
      RP(K)=DCD(K,M)
    6 DO 21 K=1,4
   21 RP(K)=RP(K)*TC
      RETURN
      END
```

```
      RXMA3=RXMA**3
      U(J)=EMV*(XPA/RXPA3-XMA/RXMA3)
      V(J)=EMV*W*(1./RXPA3-1./RXMA3)
40    CONTINUE
      RETURN
C
C       COMPUTE INFLOW VELOCITIES
C
200   XI=XR- XCB-XIZ*ELCBC
      XI=XI/ELCBC
      DO 60 J=1,IR
      W=RB(J)*.5/CD/ELCBC/RRP
65    XPA=XI+A
      XMA =XI-A
64    RXPA=SQRT(XPA*XPA+W*W)
      RXMA=SQRT(XMA*XMA+W*W)
      RXPA3=RXPA**3
       RXMA3=RXMA**3
      U(J)=EMV*(XPA/RXPA3-XMA/RXMA3)
      V(J)=EMV*W*(1./RXPA3-1./RXMA3)
      PSI=W*W/2.-EMV*(XPA/RXPA-XMA/RXMA )
60    CONTINUE
      RETURN
C
C       COMPUTE D(N) AND D-STAR(N) FOURIER COEFFICIENTD
C
3      W=0.5/CD/ELCBC
      N=IR
      CN =N
      PI =3.1415926
      DTH=PI/(CN-1.)
      TH =-DTH
      DO 80 J=1,N
      TH =TH +DTH
      CSTH =COS(TH)
      XG=0.5*(1.-CSTH)
      XI=XG-XCB-XIZ*ELCBC
      XI =XI/ELCBC
      XPA =XI+A
      XMA =XI-A
      RXPA =SQRT(XPA*XPA+W*W)
      RXMA=SQRT(XMA*XMA+W*W)
      RXPA3=RXPA**3
      RXMA3=RXMA**3
      FU(J)= EMV *( XPA/RXPA3-XMA/RXMA3)
      FV(J)=EMV*W*(1./RXPA3-1./RXMA3)
80    CONTINUE
      NOUT =6
      CALL FOURCS(FU,FU,N,NOUT,0)
      CALL FOURC((FV,FV,N,NOUT,0)
      DO 81 J=1,NOUT
      D(J)=FV(J)
      DS(J)=FU(J)
      U(J)=DS(J)
81     V(J)=D(J)
      RETURN
      END
```

```
$IBFTC 06
      SUBROUTINE CAMBER(CD,TC,RP)
C
C        SUBROUTINE TO COMPUTE INDUCED CAMBER COEFFICIENTS
C
      DIMENSION CDRAT(5),DCD(4,5),RP(4)
C
      DATA CDRAT/0.0,0.25,0.50,0.75,1.0/
      DATADCD(1,1),DCD(1,2),DCD(1,3),DCD(1,4),DCD(1,5)/0.0,0.00149,
     10.00316,0.00505,0.00696/
      DATA DCD(2,1),DCD(2,2),DCD(2,3),DCD(2,4),DCD(2,5) /0.0,0.10897,
     10.22236,0.33414,0.43940/
      DATA DCD(3,1),DCD(3,2),DCD(3,3),DCD(3,4),DCD(3,5) /0.0,0.03558,
     10.07223,0.11022,0.14928/
      DATA DCD(4,1),DCD(4,2),DCD(4,3),DCD(4,4),DCD(4,5) /0.0,-0.00203 ,
     1-0.00539,-0.00948,-0.01375/
C
      DO 2 J=1,5
      N=J
      IF(CDRAT(J)-CD)2,3,4
2     CONTINUE
    4 M=N-1
      DEL=CDRAT(N)-CDRAT(M)
      DIF=CD-CDRAT(M)
      DELTA=DIF/DEL
      DO 10 K=1,4
      RP(K)=DCD(S,M)+DELTA*(DCD(K,N)-DCD(K,M))
10    CONTINUE
      GO TO 20
  3   M=N
      DO 6K=1,4
6     RP(K)=DCD(K,M)
20    DO 21 K=1,4
 21   RP(K)=RP(K)*TC
      RETURN
      END
```

```
$IBFTC 07
      SUBROUTINE PROP(NZP,RB,BR,BTA,TCBLD,RRP,RCBRP,NZ,RA)
C
C       SUBROUTINE TO COMPUTE PROPELLER GEOMETRY PARAMETERS
C
      DIMENSION RB(25),BR(25),TCBLD(25),X(25),Y(25),Z(25),ZZ(25),BTA(25)
     1          ,RA(25)
C
      P=RCBRP*RCBRP
      D=1.-P
      X(1)=RCBRP
      ZN=NZ
      MZ=NZ+1
      DO20 J=2,MZ
      K=J-1
      AK=K
      E=AK/ZN*D+P
20    X(J)=SQRT(E)
      DO 21 J=1,NZ
21    RA(J)=X(J+1)
      X(1)=(RCBRP+RA(1))/2.
      DO 22 J=2,NZ
22    X(J)=(RA(J)+RA(J-1))/2.
      K=1
      RB(NZP+1)=RB(NZP )
      BR(NZP+1)=BR(NZP )
      BTA(NZP+1)=BTA(NZP)
      TCBLD(NZP+1)=TCBLD(NZP)
      DO 39 J=1,NZ
30    IF(RB(K)-X(J))31,32,33
32    Y(J)=BR(K)
      Z(J)=BTA(K)
      ZZ(J)=TCBLD(K)
      GO TO 39
31    K=K+1
      GO TO 30
33     DEL =X(J)-RB(K-1)
      DEL=DEL/(RB(K)-RB(K-1))
      Y(J)=DEL*(BR(K)-BR(K-1))
      Y(J)=Y(J)+BR(K-1)
      Z(J)=DEL*(BTA(K)-BTA(K-1))
      Z(J)=Z(J)+BTA(K-1)
      ZZ(J)=DEL*(TCBLD(K)-TCBLD(K-1))
      ZZ(J)=ZZ(J)+TCBLD(K-1)
39      CONTINUE
      DO 40 J=1,NZ
      RB(J)=X(J)
      BR(J)=Y(J)
      BTA(J)=Z(J)
      TCBLD(J)=ZZ(J)
40     CONTINUE
      RETURN
      END
```

```
$IBFTC SU08
      SUBROUTINE CLALF (J)
C
C      COMPUTATION OF BLADE SECTION LIFT COEFFICIENT
C
      DIMENSION TCB(10),CLMX(10)
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
C
       COMMON MZZZ,CD,RU,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON /NEAR1/NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR3/RRP,XP,Z,BLD,RB,BP,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1          RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
       COMMON/NEAR5/ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
101   FORMAT(//////////*     THE BLADE THICKNESS TO-CHORD RATIO IS OUTSID
     1E RANGE 0.0 TO .34*)
      IF(MZZZ)2,1,2
1     CONTINUE
C
C     TABLE OF VALUES OF CLMAX VERSUS BLADE THICKNESS-TO CHORD RATIO
C
      TCB(1)=0.0
      TCB(2)=0.06
      TCB(3)=0.08
      TCB(4)=0.10
       TCB(5)=0.12
       TCB(6)=0.15
      TCB(7)=0.18
      TCB(8)=C.21
      TCB(9)=0.24
      TCB(10)=0.34
C
      CLMX(1)=0.9
      CLMX(2)=0.9
      CLMX(3)=1.2
      CLMX(4)=1.45
      CLMX(5)=1.6
      CLMX(6)=1.5
       CLMX(7)=1.35
      CLMX(8)=1.3
      CLMX(9)=1.25
      CLMX(10)=1.1
      RETURN
C
 2     CONTINUE
19    IF (TCB(10)-TCBLD(J))21,20,20
20    IF(TCB(1)-TCBLD(J))22,22,21
21    PRINT 101
      NERR=1
      RETURN
```

```
 22   N=1
23    IF(TCB(N)^TCBLD(J))24,25,26
24    N=N+1
       GO TO 23
25    CLMAX=CLMX(N)
      GO TO 27
26    DEL =(TCBLD(J)-TCB(N-1))/(TCB(N)^TCB(N-1))
      CLMAX=CLMX(N-1)+DEL*(CLMX(N)^CLMX(N-1))
27    CONTINUE
      CL=2.*PI*ALPHA(J)/RAD
      STALL(J)=0.0
28    IF(CL-CLMAX)29,30,30
30     CL=CLMAX
       STALL(J)=1.0
29      CONTINUE
      RETURN
      END
```

```
$IBFTC SU09
      SUBROUTINE ARCSIN(ARG,PHI,INDEX)
C
C   SUBROUTINE FOR COMPUTING ARC SINE
C
C
C    ARG IS THE SINE OF THE ANGLE
CPHI IS THE ANGLE (PRINCIPLE VALUE ,-PI/2 TO PI/2)
C             OUTPUT FROM SUBROUTINE.
C      INDEX IS 1 IF ANGLE IS TO BE IN RADIANS.
C     INDEX IS 0 IF ANGLE IS TO BE IN DEGREES.
C
      HPI=1.5707963
      A=1.0-ARG*ARG
      IF(A)5,2,1
2     IF(ARG)4,3,3
3     PHI=HPI
      GO TO 6
4      PHI =-HPI
      GO TO 6
1     PHI=ATAN(ARG/SQRT(A))
      GO TO 6
5     WRITE(6,100)
100   FORMAT(/5X*ERROR...SIN X GREATER THAN 1.0*,//)
      GO TO 2
6     IF(INDEX)7,7,8
7     PHI=90.*PHI/HPI
8      RETURN
      END
```

```
$IBFTC SU09
      SUBROUTINE ARCSIN(ARG,PHI,INDEX)
C
C    SUBROUTINE FOR COMPUTING ARC SINE
C
C
C     ARG IS THE SINE OF THE ANGLE
CPHI IS THE ANGLE (PRINCIPLE VALUE ,-PI/2 TO PI/2)
C               OUTPUT FROM SUBROUTINE.
C      INDEX IS 1 IF ANGLE IS TO BE IN RADIANS.
C     INDEX IS 0 IF ANGLE IS TO BE IN DEGREES.
C
      HPI=1.5707963
      A=1.0-ARG*ARG
      IF(A)5,2,1
2     IF(ARG)4,3,3
3     PHI=HPI
      GO TO 6
4      PHI =-HPI
      GO TO 6
1     PHI=ATAN(ARG/SQRT(A))
      GO TO 6
5     WRITE(6,100)
100   FORMAT(/5X*ERROR...SIN X GREATER THAN 1.0*,//)
      GO TO 2
6     IF(INDEX)7,7,8
7     PHI=90.*PHI/HPI
8      RETURN
      END
```

```
$IBFTC SU10
      SUBROUTINE MATRIX (A,NR)
      DIMENSION A(6,7),B(19,19),N(12)
      NC=NR+1
      DO1 J=1,NR
1     N(J)=0
      DO 2 I=1,NR
      CON =0.
      II=I-1
      DO 3 J=1,NR
      JJ=J-1
      IF(II)4,4,5
5     DO 6 K=1,II
       IF(J-N(K))6,3,6
6     CONTINUE
4     CONA=A(J,1)
      IF (CONA)8,9,9
8     CONA=-CONA
9     IF(CONA-CON)3,3,11
11    CON=CONA
      N(I)=J
3      CONTINUE
      IF(CON)5,600,500
600   PRINT 601,I
601   FORMAT(1X,*SINGULAR, I=*,I3)
      STOP
500   NN=N(I)
      DO 12 J=1,NR
 12    A(J,NC)=0.0
      A(NN,NC)=1.0
      DIV=A(NN,1)
      DO 13 L=1,NC
13    A(NN,L)=A(NN,L)/DIV
      DO 14 L=1,NR
      IF (L-NN)15,14,15
  15  CMULT=A(L,1)
      DO 16 J=1,NC
16     A(L,J)=A(L,J)-CMULT*A(NN,J)
14    CONTINUE
      DO 17 L=1,NR
      DO 18 J=1,NR
18    A(L,J)=A(L,J+1)
17      CONTINUE
2     CONTINUE
      DO 19 J=1,NR
      DO 20 L=1,NR
      NN=N(J)
20    B(J,L)=A(NN,L)
19     CONTINUE
      DO 21 J=1,NR
      DO 22 L=1,NR
      NN=N(J)
22     A(L,NN)=B(L,J)
21    CONTINUE
      RETURN
      END
```

```
$IBFTC SU11
      SUBROUTINE FOURCS (F,B,N,NOUT,NPRINT)
C
C    SUB ) FOURCS--FOURIER COSINE SERIES DETERMINATION
C     FROM RALSTON AND WILF (-MATH. METHODS FOR DIGITAL COMPUTERS
C     CHAPT. 24 ZZ G.GOERTZEL
C
      DIMENSION F(50),FF(50),B(50)
      COMMON MZZZ
700   FORMAT(1H/)
701   FORMAT(4E15.7)
702   FORMAT(5X,*THETA*6X*FOURIER FN*6X*HORIGINAL FN*4X*FOURIER COEF*/)
      PI=3.1415927
      IF(NOUT-N)10,10,11
11     NOUT=N
10      ZRO=0.
      ONE=1.0
      TWO=2.0
      IZRO=0
      IONE =1
      ITWO =2
      HPI=1.5707963
      NH=N
      NS=ITWO*(NH-IONE)
      FNS=NS
      FR=TWO/FNS
      PIN=PI*FR
      DO 201 I=1,NH
      CJ= I-IONE
      CSI=COS(PIN*CJ)
      CCSI=CSI+CSI
      CA=ZRO
      CB=ZRO
      DO 202 J=2,NS
      JK=NS-J+ITWO
      IF(JK-NH)220,220,221
221    JK=ITWO*NH-JK
220   CC=F(JK)+CA*CCSI-CB
      CB=CA
202   CA=CC
      FF(I)=FR*(F(1)+CA*CSI-CB)
      IF(I-IONE)290,291,290
291   FF(I)=FF(I)/TWO
290    JN=NS-I-I+IONE
      IF(JN)  203,204,204
203   FF(I)=FF(I)/TWO
204   CONTINUE
201   CONTINUE
      IF(NPRINT-1)2,6,7
6      M=N
      GO TO 1
7      M=NOUT
```

```
1       CN=N
        PRINT 700
        PRINT 702
        DTH=PI/(CN-1.0)
        TH=-DTH
        DO 3 I=1,N
        TH=TH+DTH
        SUM =0.0
        DO 4 J=1,M
        CJ=J-1
4       SUM=SUM+FF(J)*COS(CJ*TH)
3       PRINT 701,TH,SUM,F(I),FF(I)
2       DO 8 K=1,NOUT
8       B(K)=FF(K)
        RETURN
        END
```

```
*IDETC SU12
      SUBROUTINE SRCRNG (CD,XSC,TC,NRR,RR,UOD)
C
C     VELOCITIES INDUCED BY DISTRIBUTION OF SOURCE RINGS
C
      DIMENSION TH(205),QD(205),PI(2),A(6),RR(25),UOD(25)
       COMMON MZ
      PI=3.1415926
      A(1)=TC*2.969
      A(2)=-TC*1.26
      A(3)=-TC*3.516
      A(4)=TC*2.843
      A(5)=-TC*1.015
      RO=200.
      DTI =200.
14     DT=PI/RO
       XS=XSC-.5
      N=RO
      NM=N
      N=N+1
      K=N+4
       DO 15 J=1,K
      TH(J)=0.
15     QD(J)=0.
      DO 19 J=2,N
      TH(J)=TH(J-1)+DT
120   T=TH(J)
      ST=SIN(T)
      SN2=SIN(T/2.)
      QD(J)=A(1)/2./SN2
      DO 18 I=2,5
      IN=I-1
      FN=IN
      INX=IN+IN-2
      FNX=INX
      QD(J)=QD(J)+FN*A(I)*(SN2**FNX)
18    CONTINUE
      QD(J)=QD(J)*ST
19    CONTINUE
      TH(N+1)=PI+1.
      QD(N+1)=QD(N)
22     QD(1)=A(1)
      TH(1)=0.
31    DTP=PI/DTI
      TP=0.
      H=DTP/3.
      UI=0.
      CST=-2.*XS
      CST2=CST*CST
40    DO 60 JR=1,NRR
      R=RR(JR)
50    K=1
      NT=1
51    DO 60 J=K,3
      CTP=COS(TP)
```

```
      ZWP=CD*(CTP-CST)
      PP=(ZWP*ZWP)+((R+1.)*(R+1.))
      PM=(ZWP*ZWP)+((R-1.)*(R-1.))
      AK2=4.*R/PP
      CALL ELLIPS (AK2,ZK,ZE)
       PP=SQRT(PP)
      U=ZWP*ZE/PP/PM
52    IF(TP-TH(NT))55,54,53
53     NT=NT+1
      GO TO 52
54    Q=QD(NT)
      GO TO 56
55     Q=QD(NT-1)+(TP-TH(NT-1))*(QD(NT)-QD(NT-1))/(TH(NT)-TH(NT-1))
56    DI(J)=Q*U
60      TP=TP+DTP
      K=2
       UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      IF(TP-PI)51,51,61
61    UV=UI*CD/PI
      UQD(JR)=UV
      TP=0.
      UI=0.
69      CONTINUE
      RETURN
       END
```

```
$IBFTC SU13
      SUBROUTINE VTXRNG (CD,XSC,IR,RP,CN,UG,XPRES,P)
C        VELOCITY INDUCED BY DISTRIBUTION OF VORTEX RINGS
C
      DIMENSION TH(205),GD(205),DI(2),RP(25),UG(25),CN(6),XPRES(25)
      DIMENSION FU(50),FV(50),CP(6),P(6,6)
C
      COMMON MZZZ
C
       C0=CN(1)
      C1=CN(2)
      C2=CN(3)
      C3=CN(4)
      C4=CN(5)
      C5=CN(6)
C
C          SET UP A TABLE OF GAMMA D/GAMMA*SIN(THETA) VERSUS THETA
C
      DG=100.
      DTI=200.
      PI=3.1415926
13    DT=PI/DG
      N=DG
      NM=N
      N=N+1
      K=N+4
      DO 12 J=1,K
      TH(J)=0.
      ST=0.0
12    GD(J)=0.
      DO 19 J=2,N
      TH(J)=TH(J-1)+DT
120   T=TH(J)
      CT=COS(T/2.)
      CT=CT/SIN(T/2.)
      A=C0*CT
      ST=SIN(T)
        B=C1*ST
      C=C2*SIN(2.*T)
       D=C3*SIN(3.*T)
      E=C4*SIN(4.*T)
      F=C5*SIN(5.*T)
      GD(J)=A+B+C+D+E+F
       GD(J)=GD(J)*ST
19    CONTINUE
      TH(N+1)=PI+1.
      GD(N+1)=GD(N)
22     GD(1)=2.*C0
      TH(1)=0.
31    DTP=PI/DTI
      TP=0.
      H=DTP/3.
      UI=0.
C
      IF(MZZZ)20,10,10
```

```
C
10    IF(IR)70,11,11
C
11    XS=XSC-.5
      CST=-2.*XS
      CST2=CST*CST
C
C       COMPUTE INFLOW VELOCITIES
C
40    DO 69 JR=1,IR
      R=RP(JR)
50    K=1
      NT=1
51    DO 60 J=K,3
      STP=SIN(TP)
      CTP=COS(TP)
      CTP2=CTP*CTP
      X2=CST2-(2.*CST*CTP)+CTP2
      X2=X2*CD*CD
       PP=X2+((R+1.)*(R+1.))
      PM=X2+((R-1.)*(R-1.))
      AK2=4.*R/PP
      CALL ELLIPS(AK2,ZK,ZE)
      PM=2.*(R-1.)/PM
      PP=1./SQRT(PP)
      U=ZK-ZE*(1.+PM)
      U=U*PP
52    IF(TP-TH(NT))55,54,53
53      NT=NT+1
      GO TO 52
54    G=GD(NT)
      GO TO 56
55    G=GD(NT-1)+(TP-TH(NT-1))*(GD(NT)-GD(NT-1))/(TH(NT)-TH(NT-1))
56    DI(J)=G*U
60    TP=TP+DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      IF(TP-PI)51,51,61
61    UG(JR)=CD/2./PI*UI
      TP=0.
      UI=0.
69     CONTINUE
      GO TO 999
C
C     COMPUTE VELOCITIES AT THE DUCT REFERENCE CYLINDER
C
70    IR=-IR
      DO 99 JR=1,IR
      X=XPRES(JR)
      K=1
       NT=1
      XS=X-.5
      CST=-2.*XS
      CST2=CST*CST
```

```
77      DO 71 J=K,3
        CTP=COS(TP)
        CTP2=CTP*CTP
        X2=CST2-(2.*CST*CTP)+CTP2
        X2=X2*CD*CD
        PP=X2+4.
        AK2=4./PP
        CALL ELLIPS(AK2,ZK,ZE)
        PP=1./SQRT(PP)
        U=(ZK-ZE)*PP
 72      IF(TP-TH(NT))75,74,73
73      NT=NT+1
        GO TO 72
 74      G=GD(NT)
        GO TO 76
 75      G=GD(NT-1)+(TP-TH(NT-1))*(GD(NT)-GD(NT-1))/(TH(NT)-TH(NT-1))
76      DI(J)=G*U
71      TP=TP+DTP
        K=2
        UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
        DI(1)=DI(3)
        IF(TP-PI)77,77,78
78      UG(JR)=CD/2./PI*UI
        TP=0.
        UI=0.
99      CONTINUE
        GO TO 999
C
C        COMPUTE F(N) AND F-STAR(N) FOURIER COEFFICIENTS
C
200     MZZ=MZZZ
        MZZZ=1
        DO 210 J=1,6
210     CP(J)=0.0
        DUM=0.0
         DO 211 L=1,6
 211      DUM=DUM+CN(L)*P(1,L)
        CP(1)=(CN(1)-DUM)/2.0
        DO 212 K=2,6
        DO 213 L=1,6
 213     CP(K)=CP(K)+CN(L)*P(K,L)
212     CP(K)=(CP(K)-CN(K))/2.0
        N=IP
        EN=N
        DTA=PI/(EN-1.)
        TA=-DTA
        DO 220  JR=1,N
        TA=TA+DTA
        CSTA=COS(TA)
        CSTA2=CSTA*CSTA
        DUM=0.0
        DO 215 K=2,6
        J=K-1
        TJ=J
215     DUM=DUM+CP(K)*COS(TJ*TA)
```

```
      FV(JR)=CP(1)+DUM
      K=1
      NT=1
 207  DO 201 J=K,3
      CTP=COS(TP)
      CTP2=CTP*CTP
      X2=CSTA2-(2.*CSTA*CTP)+CTP2
      X2=X2*CD*CD
      PP=X2+4.0
      AK2=4./PP
      CALL ELLIPS(AK2,ZK,ZE)
      PP=1./SQRT(PP)
      U=(ZK-ZE)*PP
 202  IF(TP-TH(NT))205,204,203
 203  NT=NT+1
      GO TO 202
 204  G=GD(NT)
      GO TO 206
 205   G=GD(NT-1)+(TP-TH(NT-1))*(GD(NT)-GD(NT-1))/(TH(NT)-TH(NT-1))
 206  DI(J)=G*U
 201  TP=TP+DTP
      K=2
       UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      IF (TP-PI)207,207,208
 208  FU(JR)=CD/2./PI*UI
      TP=0.0
      UI=0.0
 220   CONTINUE
      NOUT =6
      NPR=0
      CALL FOURCS(FU,FU,N,NOUT,NPR)
      DO 216 J=1,6
      UG(J)=FU(J)
 216   UG(J+6)=CP(J)
      MZZZ=MZZ
C
 999  RETURN
       END
```

```
$IBFTC SU14
       SUBROUTINE ALFRNG (CD,IR,SC,UGA,XPRES)
C      AXIAL VELOCITY INDUCED BY ALPHA VORTEX RINGS
C
       DIMENSION TH(205),GA(205),DI(3),XPRES(25),SC(6),UGA(25),ST(6)
       DIMENSION TCD(7),TSC(6,7)
       DIMENSION GS(6),F(50),HF(6),FH(5^),DVT(3)
C
       COMMON MZZZ
C
        DATA TCD/0.,0.5,0.75,1.0,1.5,2.^,5.0/
       DATA TSC(1,1),TSC(2,1),TSC(3,1),TSC(4,1),TSC(5,1),TSC(6,1) /1.-
      10.,0.,0.,^.,0./
       DATA TSC(1,2),TSC(2,2),TSC(3,2),TSC(4,2),T  (5,2),TSC(6,2) /1.
      1,-0.2382,^0.0175,0.,0.,0./
       DATA TSC(1,3),TSC(2,3),TSC(3,3),TSC(4,3),TS (5,3),TSC(6,3) /
      1,-0.345,-0.042,0.,0.,0./
       DATA TSC(1,4),TSC(2,4),TSC(3,4),TSC(4,4),T  (5,4),TSC(6,4) /
      1,-0.4151,-0.0680,0.,0.,0./
       DATA TSC(1,5),TSC(2,5),TSC(3,5),TSC(4,5),TS (5,5),TSC(6,5) /0.5
      1,-0.4834,-0.1230,-0.0121,0.,0./
       DATA TSC(1,6),TSC(2,6),TSC(3,6),TSC(4,6),TS (5,6),TSC(6,6) /0.5
      1,-0.5045,^0.1666 ,-0.0317,0.^^35,0./
       DATA TSC(1,7),TSC(2,7),TSC(3,7),TSC(4,7),TSC(5,7),TSC(6,7) /0.
      1,-^.4590,^0.26^7,-0.1246,^0.^494,0./
C
       IF(MZZZ)2,1,2
1      CONTINUE
C
C     COMPUTE SC(N) COEGFICIENTS
C
       DO 11 J=1,7
       JSC=J
       IF(CD-TCD(J))4,3,11
11     CONTINUE
3      DO 5 N=1,6
5      SC(N)=TSC(N,JSC)
       GO TO 10
4      IF(JSC-1)6,6,7
 6     RETURN
 7     DELSC= (TCD(JSC)-TCD(JSC-1))/(TCD(JSC)- CC)
       J= JSC
       DO 8 N= 1, 6
 8     SC(N)= (TSC(N,J-1)-TSC(N,J))/DELSC+ TSC(N, J)
C
C      SET UP A TABLE OF GAMMA ALPHA /V*SIN(THET^) VERSUS THETA
C
 10    PI= 3.1415926
       DG= 200.^
       DT= PI/ DG
       K= 201
       DO 12 J= 1, K
       TH(J)= 0.0
 12    GA(J)= 0.0
       DO 20 J= 2, K
```

```
      TH(J)= TH(J-1)^ DT
      T= TH(J)
      CTN= COS(T/2.0)/ SIN( T/2.0)
      CT= COS(T)
      ST(1)= SIN(T)
      ST(2)= 2.0*ST(1)*CT
      DO 13 N= 3, 5
      NM= N-1
      NMM= N-2
   13 ST(N)= 2.0*ST(NM)*CT-ST(NMM)
      GA(J)= SC(1)* CTN
      DO 14 N= 2, 6
   14 GA(J)= GA(J)+ SC(N)* ST(N-1)
      GA(J)= GA(J)+ ST(1)
   20  CONTINUE
      GA(1)= 2.^* SC(1)
      TH(K+1)= PI+ 1.0
      GA(K+1)= GA(K)
C
C     COMPUTE G STAR(N) AND H(N) FOURIER COEFFICIENTS
C
      MZZZ= 1
      DTI= 200.0
      DTP= PI/ DTI
      H= DTP/3.^
      TP= 0.0
      UI= 0.0
      VIT= 0.0
      R= 1.0
      R32= R**1.5
      RP2=  (F+1.0)**2
      N= IR
      CN= N
      DTA= PI/ (CN-1.0)
      TA= -DTA
      DO 50 JR= 1, N
      TA= TA+ DTA
      CSTA= COS( TA)
      K= 1
      NT= 1
   47 DO 41 J= K. 3
      CTP= COS(TP)
      XI= ( CTP-CSTA)* CD
      X2=XI*XI
      DEN= X2+ RP2
      AK2= 4.0*R/ DEN
C
      CALL ELLIPS( AK2, ZK, ZE)
C
      AKP= 1.0^ AK2
      AKQ= 2.^- AK2
      AK4= AK2* AK2
      AYE= ((8.0*R*AKQ/AK4)+(2.0*R)+2.^-4.0/AK2*(2.*R+1.))*ZE
      AYE= AYE+AKP/AK2*(8.*R+4.-16.*P/AK2)*ZK
      U= AYE/SQRT(DEN)/(X2+(R-1.0)*(P-1.))
```

```
      AK= SQRT( AK2)
      AKZ= 4.0* R/ RP2
      SBETA= SQRT((1.0- AKZ)/(1.0-AK2))
C
      CALL ARCSIN( SBETA, BETA, 0)
      CALL ARCSIN( AK, ASK, 0)
      CALL LAMBDA( ASK, BETA, HLMB)
C
      EPSR= 1.0
      AVT= 4.0*(ZK-ZE)/AK+2.0*PI*AK/SQRT(AKZ)*(1.0-HLMB)*
     1   SQRT((1.0-AKZ)/(AKZ-AK2))
      AVT= AVT*XI/(8.0*PI*R32)+ EPSR/4.0
 42   IF(TP- TH(NT))45, 44, 43
 43   NT= NT+ 1
      GO TO 42
 44    G=GA(NT)
      GO TO 46
 45   G= GA(NT-1)+ (TP-TH(NT-1))* (GA(NT)-GA(NT-1))/(TH(N   TH(NT-
46    DI(J)=G*U
      DVT(J)=G*AVT
41    TP =TP+DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
       VIT=VIT+H*(DVT(1)+4.*DVT(2)+DVT(3))
      DVT(1)=DVT(3)
      IF (TP-PI) 47,47,48
 48   F(JR)=-CD*UI/2./PI
      FH(JR)=CD*VIT
      TP=0.0
      UI=0.0
      VIT=0.0
50    CONTINUE
      NOUT =6
      NPR=0
      CALL FOURCS(F,F,N,NOUT,NPR)
      CALL FOURCS (FH,FH,N,NOUT,NPR)
      DO 51 J=1,NOUT
      GS(J)=F(J)
      HF(J)=FH(J)
      UGA(J+6)=HF(J)
51      UGA(J)=GS(J)
      MZZZ=0
      RETURN
  2    CONTINUE
      TP=0.0
      UI=0.0
C
C      CALCULATE THE AXIAL VELOCITY INDUCED BY THE ABOVE VORTEX DISTRIBU
C      ITION
C
7n    DO 99JR=1,IR
      X=XPRES(JR)
      K=1
      NT=1
```

```
      XS=X-.5
      CST=-2.*XS
      R=1.0
77    DO 71 J=K,3
      CTP=COS(TP)
      X2=(CTP-CST)*CD
      X2=X2*X2
      DEN=X2+(R+1.)*(R+1.)
      AK2=4.*R/DEN
      CALL ELLIPS (AK2,ZK,ZE)
      AKP=1.0-AK2
      AKQ=2.0-AK2
      AK4=AK2*AK2
      AYE=((8.0*R*AKQ/AK4)+(2.*R)+2.0-4./AK2*(2.*R+1.))*ZE
      AYE=AYE+AKP/AK2*(8.*R+4.-16.*R/AK2)*ZK
      U=AYE/SQRT(DEN)/(X2+(R-1.)*(R-1.))
72    IF (TP-TH(NT))75,74,73
73    NT=NT+1
      GO TO 72
74      G=GA(NT)
       GO TO 76
75     G=GA(NT-1)+(TP-TH(NT-1))*(GA(NT)-GA(NT-1))/(TH(NT)-TH(NT-1))
76      DI(J)=G*U
71    TP=TP +DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
       DI(1)=DI(3)
      IF (TP-PI)77,77,78
78    UGA(JR)=-CD*UI/2./PI
       TP=0.0
       UI=0.0
99    CONTINUE
999   RETURN
      END
```

```
$IBFTC SU15
      SUBROUTINE GAMCYL (CD,XC,N,RP,UG,IR,XPRES,UGP,RRP,RA)
C
C     SUBROUTINE TO CALCULATE THE AXIAL VELOCIYIES INDUCED BY CONSTANT
C             STRENGTH VOREX CYLINDERS
C
      DIMENSION  RP(25),UG(25),XPRES(25),UGP(25,25),RA(25)
      COMMON MZ
C
      PI=3.1415926
      NR=N
      CR=CD*2.
      IF(IR)2,1,2
1     XG=(XC-1.)*CR
C
C     COMPUTE THE VELOCITIES INDUCED AT THE PROPELLER STATIONC
C
20     DO 30 J=1,NR
      R=RP(J)
      IF(R)22,21,22
21    U=SQRT((XG*XG)+1.)
       U=XG/U+1.0
      U=0.5*U
      GO TO 30
22     XG2=XG*XG
      RP1=(R+1.)*(R+1.)
      RM1=(R-1.)*(R-1.)
      SNB=SQRT(XG2+RM1)
      SNB=XG/SNB
      SKG2=4.*R/(XG2+RP1)
      CALL ELLIPS (SKG2,ZK,ZE)
      SKG=SQRT(SKG2)
      CALL ARCSIN(SKG,ASKG,0)
      CALL ARCSIN(SNB,BETA,0)
            NDEX=0
      IF(BETA)23,24,24
23    BETA=-BETA
      NDEX=1
24    CALL LAMBDA(ASKG,BETA,HLMB)
      IF(NDEX)25,26,25
25    HLMB=-HLMB
      NDEX=0
26    CONTINUE
      U=.25*HLMB+.5
      TM=SKG*XG*ZK/(4.*PI)
      TM=TM/SQRT(R)
      U=U+TM
30    UG(J)=U
      GO TO 99
2     NCYL=N
C
C    COMPUTE THE VELOCITY INDUCED AT THE REFRENCE CYLINDERBY
C       THE TRAILING VORTEX CYLINDERS
C
      DO 60 J=1,NCYL
```

```
      RGR=RA(J)/RRP
      RGR2=(RGR+1.)*(RGR+1.)
      RGRM=(RGR-1.)*(RGR-1.)
      RTRGR=SQRT(RGR)
      IF (J-NCYL)31,32,32
 32   XP=1.0
      RGR=1.0
      RGR2=4.0
      RGRM=0.0
      GO TO 33
 31     XP=XC
 33   DO 60 JR=1,IR
      X=XPRES(JR)
      XG=X-XP
      XGR=XG*CR
      XGR2=XGR*XGR
      IF(XGR)38,42,38
 42   IF(RGR-1.)142,242,342
 142  UG(J)=0.0
       GO TO 60
 242  UG(J)=0.25
      GO TO 60
 342  UG(J)=0.50
      GO TO 60
 38   CONTINUE
      GKSQ=4.*RGR/( XGR2+RGR2)
      GK=SQRT(GKSQ)
      CALL ELLIPS(GKSQ,ZK,ZE)
       NDEX=0
      SNB=XGR/SQRT(XGR2+RGRM)
      CALL ARCSIN(SNB,BETA,0)
      CALL ARCSIN(GK,ASK,0)
      IF(BETA)34,35,35
  34  BETA =-BETA
      NDEX=-1
 35   CALL LAMBDA(ASK,BETA,HLMB)
      IF(NDEX)36,37,37
 36   HLMB=-HLMB
      BETA =-BETA
      NDEX=0
 37   CONTINUE
      IF(XGR)41,42,43
 43   NDEX=-1
      HLMB=-HLMB
      XGR=-XGR
 41   IF(RGR-1.)141,241,241
 141  UG(J)=GK*XGR*ZK/(4.*PI*RTRGR) -.25*HLMB
      GO TO 45
 241  UG(J)=GK*XGR*ZK/(4.*PI*RTRGR)+0.50+.25*HLMB
 45   IF(NDEX)46,60,60
```

```
46      XGR=-XGR
        NDEX=0
        HLMB=-HLMB
47      IF(RGR-1.)147,247,347
147      UG(J)=-UG(J)
        GO TO 60
247      UG(J)=-UG(J)+0.5
        GO TO 60
347      UG(J)=-UG(J)+1.0
60      UGP(J,JR)=UG(J)
99      RETURN
        END
```

```
$IBFTC SU16
      SUBROUTINE BNCOEF (C,BGI)
C
C       FOURIER ANALYSIS OF RADIAL VELOCITY INDUCED BY AN
C          ACTUATOR DISK IN THE EXIT PLANE--B(N)
C
       DIMENSION BG(6,2),BGI(6),BGO(6)
      COMMON MZZZ
      PI =3.1415927
      MZZZ=-10
      N=100
      KMAX=6
       DO 10 K=1,KMAX
10    BGI(K)=0.0
      EPS =PI/180.
      PIM =PI-EPS
      CN=N
      DELTH =.5*PIM/CN
       NP=N+1
      DO 6 I=1,NP
       DO 3J=1,2
       FI=I-2
      FJ=J
        TH=FI*PIM/CN+FJ*DELTH
      STH=SIN(TH)
      CTH=COS(TH)
72    XG=-0.5*C*(1.+CTH)
73    XGSQ=XG*XG
      GKSQ=XGSQ/(4.+XGSQ)
      GK=SQRT(4./(4.+XGSQ))
      CALL ELLIPS (GKSQ,ZKG,ZCG)
      BGC=.5/PI*((GK-2./GK)* ZKG+2./GK*ZCG)
      STHK=0.
        CTHK=1.
      DO 2 K=1,KMAX
      BG(K,J)=BGC*CTHK
      CON=STHK*CTH+CTHK*STH
      CTHK=CTHK*CTH-STHK*STH
2      STHK=CON
3     CONTINUE
      DO 5 K=1,KMAX
       IF(I-1)4,5,4
4      BGI(K)=BGI(K)+DELTH*(BG(K,1)+(BGO(K)+BG(K,1)+BG(K,2))/3.)
5     BGO(K)=BG(K,2)
6      CONTINUE
       EXTRA =-EPS/PI*ALOG(4./EPS*SQRT(2./C))
      SI=-1
      DO 7 K=1,KMAX
      SI=-SI
7      BGI(K)=2.*(BGI(K)+SI*EXTRA)/PI
      BGI(1)=.5*BGI(1)
      MZZZ=1
       RETURN
      END
```

```
$IBFTC SU17
      SUBROUTINE ANCOEF(NCYL,N,RA,XP,CD,RRP,NRUN,NPRINT BS,A,AS)
      DIMENSION F(50),BS(6),A(25,6),AS(25,6),RA(25)
      COMMON MZZZ
142   FORMAT(1H1,4X,*RUN NUMBER*,I5,49X,*PAGE*,I3,2H-F,//)
242   FORMAT(10X,*FOURIER COSINE SERIES COEFFICIENTS--OUTPUT INDEX =*,
     1 I3)
       NPAGE =0
      NOUT =6
 20   CR=CD*2.0
      PI=3.1415927
21     CN=N
      DTH=PI/(CN-1.)
      TH=-DTH
C
C      COMPUTE B-STAR(N)  FOURIER COEFFICIENTS
C
22     DO 30 J=1,N
       TH=TH+DTH
      CSTH=COS(TH)
       XG=-.5*CR*(1.+CSTH)
      GKSQ=XG*XG
       GKSQ=4.0+GKSQ
       GKSQ=4.0/GKSQ
       CALL ELLIPS(GKSQ,ZK,ZE)
       GK=SQRT(GKSQ)
30    F(J)=0.25+(GK*XG/4.*ZK/PI)
       IF(NPRINT)171,171,170
170     NPAGE =NPAGE+1
       PRINT 142,NRUN,NPAGE
       PRINT242,NPRINT
171   CALL FOURCS (F,F,N,NOUT,NPRINT)
      DO 31 J=1,NOUT
31    BS(J)=F(J)
C     COMPUTE A(N)  FOURIER COEFFICIENTS
C
32    DO 40 J=1,NCYL
      RRA=RRP/RA(J)
      RRAP=RRA+1.0
      RRAP=RRAP*RRAP
      RRA4=RRA*4.0
      RASQT=SQRT(RRA)
       TH=-DTH
33     DO 38 K=1,N
       TH=TH+DTH
       CSTH=COS(TH)
      XR=CR*(1.-CSTH)*.5
      XG=XR-(XP*CR)
       XG=XG*RRA
       XGSQ =XG*XG
      GKSQ=RRAP+XGSQ
       GKSQ=RRA4/GKSQ
      GK=SQRT(GKSQ)
       CALL ELLIPS (GKSQ,ZK,ZE)
      F(K)=((ZE-ZK)/GK+(GK*ZK/2.))/PI/RASQT
```

```
38    CONTINUE
      IF(NPRINT)173,173,172
172    NPAGE=NPAGE+1
       PRINT 142,NRUN,NPAGE
      PRINT 242,NPRINT
173    CALL FOURCS(F,F,N,NOUT,NPPINT)
      DO 39 K=1,NOUT
39    A(J,K)=F(K)
40    CONTINUE
C
C        COMPUTE A-STAR(N) FOURIER COEFFICIENTS
C
42     DO 50 J=1,NCYL
      RRA=RRP/RA(J)
       RPRIM= 1.0/RRA
      RRAP=RRA+1.0
      RRAP=RRAP*RRAP
      RRA4=4.0*RRA
      RASQT=SQRT(RRA)
      TH=-DTH
43    DO 48 K=1,N
       TH=TH+DTH
      CSTH=COS(TH)
      XR=CR*(1.0-CSTH)*.5
      XG=XR-(XP*CR)
      XG=XG*RRA
       XGSQ=XG*XG
      GKSQ=RRAP+XGSQ
      GKSQ=RRA4/GKSQ
       GK=SQRT(GKSQ)
      CALL ELLIPS(GKSQ,ZK,ZE)
      NDEX=0
       SNB=RRA-1.0
      SNB=SNB*SNB
       SNB=XGSQ+SNB
      SNB=1.0/SQRT(SNB)
       SNB=XG*SNB
      CALL ARCSIN(SNB,BETA,0)
      CALL ARCSIN(GK,ASK,0)
      XG=XG*RPRIM
      IF(BETA)62,63,63
62    BETA=-BETA
      NDEX=-1
63     CALL LAMBDA(ASK,BETA,HLMB)
      IF(NDEX)64,65,65
64     HLMB=-HLMB
       NDEX=0
      BETA=-BETA
65     CONTINUE
       IF(XG)71,72,73
72    IF(RPRIM-1.0)74,75,76
74    F(K)=0.0
      GO TO 48
75    F(K)=-0.25
      GO TO 48
```

```
76      F(K)=-0.50
        GO TO 48
73      NDEX=-1
        HLMB=-HLMB
        XG=-XG
71      IF(RPRIM-1.0)77,78,78
77      F(K)=-GK*XG*ZK/4.0/PI*RASQT
        F(K)=F(K)^(.25*HLMB)
        GO TO 79
78      F(K)=-GK*XG*ZK/4.0/PI*RASQT
        F(K)=F(K)-.5-(.25*HLMB)
79      IF(NDEX)80,48,48
80      XG=-XG
        NDEX=0
        IF(RPRIM-1.)81,82,83
81      F(K)=-F(K)
        GO TO 48
82      F(K)=-F(K)-0.5
        GO TO 48
83      F(K)=-F(K)-1.0
48      F(K)=-F(K)
        IF(NPRINT)175,175,174
174     NPAGE =NPAGE+1
        PRINT142,NRUN,NPAGE
        PRINT242,NPRINT
175     CALL FOURCS(F,F,N,NOUT,NPRINT)
        DO 49 K=1,NOUT
49      AS(J,K)=F(K)
50      CONTINUE
        RETURN
        END
```

```
$IBFTC SU18
      SUBROUTINE CNCOEF(GV,XC)
      DIMENSION A(6),B(6),AS(6),BS(6),CN1(6),CN2(6),Q(6,7),P(6,6),XC(6)
      DIMENSION CVG(6)
      COMMON MZZZ,CD,RO,R1,R2,R3,PI,B,BS,A,AS,P
      VG=1.0/GV
      C=2.*CD
C
C           CALCULATE COEFFICIENT  MATRIX
C
      TRO=2.*RO
35    C4D=C/8.0
36    CLN=ALOG(32.0/C)-1.0
      Q(1,1)=1.0-P(1,1)-TRO*C4D*CLN-R1*C4D
      Q(1,2)= R2/4.0*C4D-TRO*C4D*0.5*CLN
      Q(1,3)=-P(1,3)-R1*0.5*C4D+R3*C4D/6.0
      Q(1,4)=-R2*C4D/4.0
      Q(1,5)=-P(1,5)-R3*C4D/6.0
      Q(1,6)=0.0
      Q(2,1)=P(2,1)-TRO*C4D-2.*R1*C4D*CLN-R2*C4D
      Q(2,2)=P(2,2)-1.0-R1*C4D*CLN-(R1-R3)*C4D/4.0
      Q(2,3)=C4D*(-RO-R2/3.)
      Q(2,4)=P(2,4)-C4D*(R3-R1+R1)/8.0
      Q(2,5)=-R2*C4D/6.0
      Q(2,6)=P(2,6)-R3*C4D/8.0
      Q(3,1)=P(3,1)-C4D*(R1+R3)-2.*R2*C4D*CLN
      Q(3,2)=RO*C4D/2.0-R2*C4D*CLN
        Q(3,3)=P(3,3)-1.0-C4D*(R1/3.+R3/2.)
      Q(3,4)=C4D*(R2/8.-RO/2.)
      Q(3,5)=P(3,5)+C4D*(R3/10.-R1/6.)
      Q(3,6)=-R2*C4D/8.0
      Q(4,1)=P(4,1)-C4D*(R2+2.*R3*CLN)
      Q(4,2)=P(4,2)+C4D*(R1/4.-R3*CLN)
       Q(4,3)=C4D*(RO/3.-R2/2.)
      Q(4,4)=P(4,4)-1.0- R1*C4D/8.0
      Q(4,5)=C4D*(R2/10.-RO/3.)
      Q(4,6)=P(4,6)-R1*C4D/8.0
      Q(5,1)=P(5,1)-R3*C4D
      Q(5,2)=R2*C4D/4.0
      Q(5,3)=P(5,3)+C4D*(R1/6.-R3/2.)
      Q(5,4)=C4D*( RO/4.-R2/4.)
       Q(5,5)=P(5,5) -1.0-C4D*R1/15.
      Q(5,6)=-RO*C4D/4.0
      Q(6,1)=P(6,1)
      Q(6,2)=P(6,2)+R3*C4D/4.
      Q(6,3)=C4D*R2/6.0
      Q(6,4)=P(6,4)+ C4D*(R1/8.-R3/4.)
      Q(6,5)=C4D*(RO/5.-R2/6.)
       Q(6,6)=P(6,6)-1.0-C4D*R1/8.
C
C       CALCULATE CONSTANTS
C
       DO 40 I=1,6
      CN1(I)=TRO*BS(I)-(2.*B(I))
      GO TO (41,42,43,44,45,46),I
```

```
41     CN1(I)=CN1(I)+(R1*BS(2))+(R2*BS(3))+(R3*BS(4))
       GO TO 40
42     CN1(I)=CN1(I)+R1*(BS(3)+(2.*BS(1)))+R2*(BS(2)+BS(4))+R3*(BS(3)+BS(
      1(5))
       GO TO 40
43     CN1(I)=CN1(I)+R1*(BS(2)+BS(4))+R2*(BS(5)+(2.*BS(1)))+R3* (BS(2)+BS
      1(6))
       GO TO 40
44     CN1(I)=CN1(I)+R1*(BS(3)+BS(5))+R2*(BS(2)+BS(6))+2.*R3*BS(1)
       GO TO 40
45     CN1(I)=CN1(I)+R1*(BS(4)+BS(6))+R2*BS(3)+R3*BS(2)
       GO TO 40
46     CN1(I)=CN1(I)+R1*BS(5)+R2*BS(4)+R3*BS(3)
40     CONTINUE
       AS(1)=AS(1)+1.0
       DO 50 I=1,6
       CN2(I)=-2.*(A(I))+(TRU*(AS(I)))
       GO TO (51,52,53,54,55,56),I
51     CN2(I)=CN2(I)+R1*(AS(2))+R2*(AS(3))+R3*(AS(4))
       GO TO 50
52     CN2(I)=CN2(I)+R1*(2.*(AS(1))+AS(3))+R2*(AS(2)+AS(4))+R3*(AS(3)+AS(
      15))
       GO TO 50
53     CN2(I)=CN2(I)+R1*(AS(2)+AS(4))+R2*(2.*(AS(1))+AS(5))+R3*(AS(2)+AS
      1(6))
       GO TO 50
54     CN2(I)=CN2(I)+R1*(AS(3)+AS(5))+R2*(AS(2)+AS(6))+R3*2.*(AS(1))
       GO TO 50
55     CN2(I)=CN2(I)+R1*(AS(4)+AS(6))+R2*(AS(3))+R3*(AS(2))
       GO TO 50
56     CN2(I)=CN2(I)+R1*(AS(5))+R2*(AS(4))+R3*(AS(3))
50     CONTINUE
       AS(1)=AS(1)-1.0
C
       CALL MATRIX(Q,6)
59     DO 60 I=1,6
       XC(I)=0.0
60     CVG(I)=CN1(I)+CN2(I)/GV
       DO 61 I=1,6
       DO 61 J=1,6
61     XC(I)=XC(I)+Q(I,J)*CVG(J)
       RETURN
       END
```

```
$IBFTC SU19
      SUBROUTINE PRESS
C
      DIMENSION TCRAT(7),XV(19),DVA(8,7),F(8,19),UP(25),UM(25),SUMU(25)
      DIMENSION ST(5),UCBP(25),VCBP(25),CST(5),SAVU(25),FX(25)
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25),
     1     UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
C
      COMMON MZZZ,CD,RU,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON /NEAR1/NRUN,NOLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAJ,NPHI,NPRINT
      COMMON/NEAR2/ C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/RRP,XR,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1          RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON /NEAR4/UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,T LK
C
      DATA TCRAT/0.06,0.08,0.12,0.15,0.18,0.21, .24/
      DATA XV/0.0,0.005,0.0125,0.025,0.05,0.075,0.10,0.15,0.2,0.25,0.3,
     1     0.4,0.5,0.6,0.7,0.8,0.9,0.95,1.0/
      DATA DVA(1,1),DVA(1,2),DVA(1,3),DVA(1,4),DVA(1,5),DVA(1,6),DVA(1,7
     1) /3.992,2.015,1.364,0.984,0.696,0.462,0.478/
      DATA DVA(2,1),DVA(2,2),DVA(2,3),DVA(2,4),DVA(2,5),DVA(2,6),DVA(2,7
     1) /2.90,1.795,1.31,0.971,0.694,0.561,0.478/
      DATA DVA(3,1),DVA(3,2),DVA(3,3),DVA(3,4),DVA(3,5),DVA(3,6),DVA(3,7)
     1) /1.988,1.475,1.199,0.934,0.685,0.558,0.478/
      DATA DVA(4,1),DVA(4,2),DVA(4,3),DVA(4,4),DVA(4,5),DVA(4,6),DVA(4,7
     1) /1.60,1.312,1.112,0.90,0.675,0.557,0.478/
      DATA DVA(5,1),DVA(5,2),DVA(5,3),DVA(5,4),DVA(5,5),DVA(5,6),DVA(5,7
     1) /1.342,1.178,1.028,0.861,0.662,0.555,0.478/
      DATA DVA(6,1),DVA(6,2),DVA(6,3),DVA(6,4),DVA(6,5),DVA(6,6),DVA(6,7
     1) /1.167,1.065,0.946,0.818,0.648,0.550,0.478/
      DATA DVA(7,1),DVA(7,2),DVA(7,3),DVA(7,4),DVA(7,5),DVA(7,6),DVA(7,7
     1) /1.050, .964,0.870,0.771,0.632,0.542,0.478/
      DATA F(1,1),F(1,2),F(1,3),F(1,4),F(1,5),F(1,6),F(1,7),F(1,8),F(1,9
     1),F(1,10),F(1,11),F(1,12),F(1,13),F(1,14),F(1,15),F(1,16),F(1,17),
     2F(1,18),F(1,19) /0.,0.938,1.057,1.089,1.103,1.107,1.101,1.098 ,
     1  1.091,1.086,1.078,1.066,1.053,1.042,1.028,1.013,0.990,0.974,0./
      DATA F(2,1),F(2,2),F(2,3),F(2,4),F(2,5),F(2,6),F(2,7),F(2,8),F(2,9
     1),F(2,10),F(2,11),F(2,12),F(2,13),F(2,14),F(2,15),F(2,16),F(2,17),
     2F(2,18),F(2,19) /0.,0.890,1.050,1.105,1.128,1.133,1.130,1.128,
     1  1.122,1.114,1.106,1.089,1.072,1.054,1.039,1.017,0.934,0.969,0./
      DATA F(3,1),F(3,2),F(3,3),F(3,4),F(3,5),F(3,6),F(3,7),F(3,8),F(3,9
     1),F(3,10),F(3,11),F(3,12),F(3,13),F(3,14),F(3,15),F(3,16),F(3,17),
     2F(3,18),F(3,19) /0.,0.8 0,1.  5,1.114,1.174,1.184,1.138,1.188,
     1  1.183,1.174,1.162,1.135,1.1 8,1.080,1.053,1.022,0.978, .952,0./
      DATA F(4,1),F(4,2),F(4,3),F(4,4),F(4,5),F(4,6),F(4,7),F(4,8),F(4,9
     1),F(4,10),F(4,11),F(4,12),F(4,13),F(4,14),F(4,15),F(4,16),F(4,17),
     2F(4,18),F(4,19) / 0.,0.739,0.966,1.112,1.204,1.224, .233,1.233,
     1  1.229,1.218,1.204,1.170,1.131,1.098,1.064,1.024,0.972,0.934,0./
      DATA F(5,1),F(5,2),F(5,3),F(5,4),F(5,5),F(5,6),F(5,7),F(5,3),F(5,9
```

```
     1),F(5,10),F(5,11),F(5,12),F(5,13),F(5,14),F(5,15),F(5,16),F(5,17),
     2F(5,18),F(5,19) /0.,0.682,0.926,1.103,1.228,1.264,1.276,1.278,
     1  1.275,1.262,1.247,1.205,1.154,1.116,1.074,1.025,0.966,0.914,0./
      DATA F(6,1),F(6,2),F(6,3),F(6,4),F(6,5),F(6,6),F(6,7),F(6,3),F(6,9
     1),F(6,10),F(6,11),F(6,12),F(6,13),F(6,14),F(6,15),F(6,16),F(6,17),
     2F(6,18),F(6,19) /0.,0.630,0.887,1.087,1.242,1.297,1.317,1.325,
     1  1.320,1.306,1.290,1.240,1.178,1.133,1.085,1.027,0.957,0.395,0./
      DATA F(7,1),F(7,2),F(7,3),F(7,4),F(7,5),F(7,6),F(7,7),F(7,8),F(7,9
     1),F(7,10),F(7,11),F(7,12),F(7,13),F(7,14),F(7,15),F(7,16),F(7,17),
     2F(7,18),F(7,19) /0.,0.579,0.848,1.063,1.244,1.322,1.354,1.374,
     1  1.368,1.350,1.333,1.277,1.204,1.151,1.097,1.032,0.944,0.879,0./
C
700    FORMAT(///10X,*DUCT PRESSURE DISTRUBUTION CALCUTION ASSUMES T/C
     1=*,F5.3//)
C
       IF(MZZZ)2,1,2
1      CONTINUE
C
CSET UP A TABLE OF CONTINUOUS VELOCITU CORRECTION FACTORS
C
       DO 31 J=1,7
       TCP=TCRAT(J)
       JTC=J
       IF(TC-TCRAT(J))32,33,31
32     IF(J-1)132,132,34
31     CONTINUE
132     PRINT 700, TCP
33     DO 38 N=1,19
        IF (N-7)37,37,38
37     DVA(8,N)=DVA(JTC,N)
38     F(8,N)=F(JTC,N)
       GO TO 39
34     DELTC=(TCRAT(JTC)-TCRAT(JTC-1))/(TCRAT(JTC)-TC)
       J=JTC
       DO 35 N=1,19
35     F(8,N)=(F(J-1,N)-F(J,N))/DELTC+F(J,N)
C
C          SET UP A TABLE OF THE DISCONTINUOUS VELOCITY CORRECTION FAACTORX
C
       DO 36 N=1,7
36     DVA(8,N)=(DVA(J-1,N)-DVA(J,N))/DELTC+DVA(J,N)
39     CONTINUE
       RETURN
C
2      CONTINUE
C
C         LOOK UP CONTINUOUS VELOCITY CORRECTION FACTORS
C
       DO 49 N=1,IR
       DO 42 J=1,19
       JK=J
       IF(XPRES(N)-XV(J))41,47,42
42     CONTINUE
41     DELX=(XV(JK)-XV(JK-1))/( XV(JK)-XPRES(N))
       FX(N)=(F(8,JK-1)-F(8,JK))/DELX+F(8,JK)
```

```
       GO TO 49
47    FX(N)=F(8,JK)
49     CONTINUE
      SNALF=SIN(ALF/RAD)
      CSALF=COS(ALF/RAD)
C
C      COMPUTE VELOCITY INDUCED BY TRAILING VORTEX CYLINDERS
C
      CALL GAMCYL(CD,XP,NZ,RB,UG,IR,XPRES,UGP,RRP,RA)
      DO 10 J=1,IR
10    SUMU(J)=1.0
      DO 12 J=1,IR
      DO 12 N=1,NZ
12    SUMU(J)=SUMU(J)+UGP(N,J) *GV(N)
C
C     COMPUTE VELOCITY INDUCED BY THE DUCT BOUNC VORTICITY
C
      IRC=-IR
      CALL VTXRNG (CD,XP,IRC,RB,C,UGD,XPRES,P)
      DO 14 J=1,IR
14    SUMU(J)=SUMU(J)+UGD(J)*GV(NZ)
C
C        COMPUTE VELOCITY INDUCED BY THE CENTRE BODY
C
       IRC=-IR
      CALL HUB (CD,XR,XCB,ELCDC,RMAX,IRC,XPRES,RB,RRP,UCBP,VCBP)
      DO 16 J=1,IR
       SUMU(J)=SUMU(J)+UCBP(J)*CORCB
      SUMU(J)=SUMU(J)*CSALF
16    SAVU(J)=SUMU(J)
      IF( SNALF) 20, 21,20
C
C     COMPUTE VELOCITY INDUCED BY ALPHA VORTEX RINGS
C
 20   CALL ALFRNG( CD, IR, SC, UGA, XPRES)
      GO TO 23
C
 21   DO 22 J= 1, IR
 22   UGA(J)= 0.0
 23   DO 99 M= 1, NPHI
      CSPHI= COS(PHI(M)/RAD)
      SNPHI= SIN(PHI(M)/RAD)
      DO 90 J= 1, IR
 90   SUMU(J)= SAVU(J)
      DO 25 J= 1, IR
 25   SUMU(J)= SUMU(J)+ UGA(J)* SNALF*CSPHI
 30   CONTINUE
C
C     CORRECT CONTINUOUS INDUCED VELOCITY
C
      DO 48 N= 1, IR
 48   SUMU(N)= SUMU(N)* FX(N)
C
C     COMPUTE DISCONTINUOUS PORTION OF SURFACE X  VELOCITY
C
```

```
$IBFTC SU20
      SUBROUTINE OUTPUT
      DIMENSION B(6), BS(6), SA(6), SAS(6), P(6, 6)
      DIMENSION C(6), A(25, 6), AS(25, 6), D(6), DS(6), SC(6), GS(6), H(
     1     6)
      DIMENSION RB(25), BR(25), BTA(25), TCBLD(25), RA(25), XPRES(25),
     1     PHI(5)
      DIMENSION UG(25), UQD(25), UGD(25), UV(25), UCB(25), VCB(25),
     1 UGP(25,25), UGA(25), GV(25), GRV(25), CPP(5, 25), CPM(5, 25)
      DIMENSION ALPHA(25), STALL(25), JSTL(25), TALK(20)
      DIMENSION CTDP(10), CNDP(5), CMDP(5), E(6),  ES(6), F(6), FS(6)
C
      COMMON MZZZ,CD, RU, R1,R2, R3, PI, B, BS, SA, SAS, P
      COMMON/ NEAR1/ NRUN,NBLD, NZ, MZ, NPRES, IR, NTIME, NERR, NPAG,
     1  NPHI,  NPRINT
      COMMON/NEAR2/ C,A,AS, D, DS, SC, GS, H
      COMMON/ NEAR3/ RRP, XP, Z, BLD, RB,BR, BTA,TCBLD,TC,RCBRP, APA,
     1   ALF,XPRES,RA,XCB,XR,ELCBC,RMAX,PHI, CORJ,CORCB
      COMMON/ NEAR4/ UG, UQD, UGD, UV, UCB, VCB, UGP, UGA,GV, GAM,
     1   GRV, CPP, CPM
      COMMON/ NEAR5/ ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
 101  FORMAT(*RUN NUMBR*,I5,49X,,*PAGE*,I3//)
 701  FORMAT(*RUN NUMBER*,I5,,*OPTIONAL OUTPUT*,17X,*PAGE*,I3//)
702   FORMAT(/17X1H(,F5.2,1H),29X1H(,F5.2,1H))
703   FORMAT(1H1,5X, *RUN NUMBER*,I5,10X,*DUCT SURFACE PRESSURE DISTRIBU
     1TION*,10X, *PAGE*,I3//)
102    FORMAT(5X,20A4//)
113   FORMAT(9X,4HH(N))
114   FORMAT(9X5HSC(N))
115   FORMAT(9X,4HC(N))
116    FORMAT(9X,4HB(N))
117   FORMAT(9X,5HB(N)*)
118   FORMAT(9X,8HSUM A(N))
119   FORMAT(9X,9HSUM A(N)*)
120   FORMAT(9X,4HD(N))
121   FORMAT(9X5HD(N)*)
122   FORMAT(9X6HA(M,N))
123   FORMAT(9X7HA(M,N)*)
124   FORMAT(14X,*EFFECTIVE CAMBER *,4F10.6//)
143   FORMAT(/10X,*FOURIER COSINE SERIES COEFFICIENTS*,/15X5HN=1,65X4HM=
     11,I2//)
148    FORMAT(7X,*DUCT...  C/D*,6X4HXP/C6X3HT/C6X6HRTE/RP4X6HRCB/RP5X4HA
     1P/A/10X6F10.6//)
149   FORMAT(15X5HALPHA,5X1HJ,9X2HJ',6X8HJ COS(A),2X8HJ'COS(A)/
     110X,F10.3,4F10.5)
150   FORMAT(5X1HN4X4HR/RP4X5HUQD/V7X5HUGD/V7X4HUG/V8X5HUCE/V6X8HGAMMA/R
     1V)
151   FORMAT(///15XI5,2X,*ITERATIONS    EPSILON =*,F9.6)
152   FORMAT(/17X6HINFLOW,19X5HBLADE/5X1HN4X4HR/RP5X3HU/V8X5HGAM/V7X5HAL
     1PHA6X9HDELTA P/Q)
153   FORMAT(/9X6HCTP(D)5X6HCTD(P)4X7HCTD(P),5X4HCTDP7X5HCTDP,6X4HCNDP,7
     1X4HCMDP)
154   FORMAT(2X4H(A) ,7(1PE11.4))
155   FORMAT(2X4H(B) ,7(1PE11.4))
```

```
156   FORMAT(///5X8HNOTES...//10X*(A)  COEFFICIENTS BASED ON FREE STREAM
     1 DYNAMIC PRESSURE//*//10X*(B)  COEFFICIENTS BASED ON PROPELLER TIP
     1 SPEED*)
157   FORMAT(/11X,*    BLADE SECTION LIFT COEFFICIENT HAS EXCEEDED CLMAX
     1 */20X, *IN ANNULI NOS.*,25I3)
244   FORMAT(5X,6(1PE13.6))
245   FORMAT(F9.4,5(1X2(1PE10.3)))
250   FORMAT(//2X7HAZIMUTH/3X8HPHI =  /,5(3H---3X,F7.2,4X4H---/))
251   FORMAT(/5X4HX/C  ,1(3X6HCP(IN),4X7HCP(OUT),1X))
252   FORMAT(/5X4HX/C  ,2(3X6HCP(IN),4X7HCP(OUT),1X))
253   FORMAT(/5X4HX/C  ,3(3X6HCP(IN),4X7HCP(OUT),1X))
254   FORMAT(/5X4HX/C  ,4(3X6HCP(IN),4X7HCP(OUT),1X))
255   FORMAT(/5X4HX/C  ,5(3X6HCP(IN),4X7HCP(OUT),1X))
256   FORMAT(///9X*PRESSURE COEFFICIENTS BASED ON PROPELLER TIP SPEED*)
257   FORMAT(//9X*PRESSURE COEFFICIENTS BASED ON FREE STREAM VELOCITY*)
260   FORMAT(I6,F9.5,5(1PE12.5))
261   FORMAT(2X1H*I3,F9.5,5(1PE12.5))
C
      IF(NTIME -50) 930, 930, 931
931   NERR= 1
      NPRINT= NPRINT+1
930   SNALF= SIN(ALF/ RAD)
      CSALF= COS( ALF/ RAD)
      ARJV= ARJ/ CSALF
      ARJVP= ARJP/ CSALF
      NCYL= MZ
C
      IF(NPRINT. EQ. 1.OR.NPRINT.GE.11) GO TO 900
      GO TO 901
900   NPAG= NPAG+ 1
      WRITE(6, 701) NRUN, NPAG
      WRITE (6, 102) TALK
      WRITE(6, 148) CD,XP, TC,RRP, RCBRP, APA
      WRITE(6, 124) R0, R1, R2, R3
      WRITE( 6, 149) ALF, ARJV, ARJVP, ARJ, ARJP
      WRITE(6, 702) CORJ, CORCB
      WRITE(6, 150)
      DO 61 J= 1, NZ
      X= UG(J)* GAM
      Y= UGD(J)* GAM
      R= RB(J)* RRP
      XQ= UQD(J)* CORJ
      XY= UCB(J)* CORCB
61    WRITE (6, 260) J,R,XQ,Y, X,XY,GRV(J)
      WRITE(6, 151) NTIME, EPS
901   IF(NPRINT.GE. 10)  GO TO 902
      GO TO 903
902   NPAG= NPAG+ 1
      WRITE(6, 701) NRUN, NPAG
      WRITE (6, 102) TALK
      WRITE (6,148)CD,XP,TC,RRP, RCBRP, APA
      WRITE (6,149) ALF,ARJV,ARJVP,ARJ,ARJP
      WRITE (6, 143)NCYL
      WRITE(6, 115)
      WRITE(6, 244) (C(J), J= 1, 6)
```

```
      IF( ALF) 904, 905,904
 904  WRITE(6, 114)
      WRITE (6, 244) ( SC(J), J= 1, 6)
 905   CONTINUE
      WRITE(6, 116)
      WRITE( 6, 244)(B(K), K= 1, 6)
      WRITE ( 6, 117)
      WRITE( 6, 244) ( BS( K), K= 1, 6)
      WRITE( 6, 118)
      WRITE( 6, 244) ( SA(J), J= 1, 6)
      WRITE( 6, 119)
      WRITE(6, 244) ( SAS(J), J= 1, 6)
      WRITE(6, 120)
      WRITE( 6, 244) (D(N), N= 1, 6)
      WRITE(6, 121)
      WRITE(6,244) (DS(N), N= 1, 6)
      WRITE(6, 113)
      WRITE( 6, 244) (H(N), N= 1, 6)
      WRITE(6, 122)
      DO 924 M= 1, NCYL
 924  WRITE(6, 244) (A(M,N), N= 1, 6)
      WRITE (6, 123)
      DO 925 M= 1, NCYL
 925  WRITE(6, 244) (AS(M,N),N= 1, 6)
C
 903  NPAG= NPAG+ 1
      WRITE(6, 101) NRUN, NPAG
      WRITE(6, 102) TALK
      WRITE(6, 148) CD, XP, TC, RPP, RCBRP, APA
      WRITE( 6, 149) ALF, ARJV, ARJVP, ARJ, ARJP
      WRITE(6, 152)
      X= 0.0
      Y= 0.0
      DO 63 J= 1, NZ
      X= GRV(J)/PI/ARJP*BLD/Z
      Y= Y+ X
      R= RB(J)* RPP
      DELP= GRV(J)* BLD/PI/ ARJP
      IF( STALL(J)) 62, 64, 62
 64   WRITE(6, 260) J,R,UV(J),GV(J), ALPHA(J), DELP
      GO TO 63
 62   WRITE(6, 261) J, R,UV(J), GV(J), ALPHA(J), DELP
 63   CONTINUE
      X= 0.0
C
C     COMPUTE  DUCTED PROPELLER THRUST COEFFICIENTS
C
      CTCF= ARJ*ARJ*PI/APA/8.0/CSALF/CSALF
      CTDP(1)= Y*APA*CSALF*CSALF
      DO 20 J= 1, 6
 20   E(J)= SA(J)+D(J)*CORCB+B(J)* GAM
      CON= -CD*PI*GAM*CSALF* CSALF
      CTDP(2)= C(1)*(4.0*E(1)+2.*E(2))-2.*C(2)*E(1)
      CTALF= SC(1)*(4.*H(1)+ 2.0*H(2))- 2.0* SC(2)* H(1)
      DO 21 N= 2, 5
```

```
      CTALF= CTALF+ SC(N+1)* H(N)- SC(N)* H(N+ 1)
   21 CTDP(2)= CTDP(2)+ C(N+1)*E(N) -C(N)* E(N+1)
      CTDP(2)= CTDP(2)*CON+PI*CD*(2.0*SC(1)+ SC(2))*SNALF*SNALF
      CTALF= -PI*CD/2.0*SNALF*SNALF* CTALF
      CTDP(2)= CTDP(2)+ CTALF
      X= RRP* RRP
      X= 1.0-(1.0/X)
      X= X* DELP* CSALF* CSALF
      CTDP(3)= CTDP(2)+ X
      CTDP(4)= CTDP(1)+ CTDP(2)
      CTDP(5)= CTDP(1)+ CTDP(3)
      DO 22 J= 1, 5
   22 CTDP(J+ 5)= CTDP(J)* CTCF
C
C     COMPUTE DUCT NORMAL FORCE COEFFICIENTS
C
      DO 30 J=1,5
      CMDP(J)=0.0
   30 CNDP(J)=0.0
      SACA=SNALF*CSALF
      IF(SACA)31,23,31
   31  MZZZ=-1
C
C        COMPUTE F(N)  AND F-STOR(N) FOURIER COEFFICIENTS
C
      NF=50
      CALL VTXRNG(CD,XP,NF,RB,C,UGD,XPRES,P)
      MZZZ=1
       DO 32 J=1,6
      FS(J)=UGD(J)
   32 F(J)=UGD(J+6)
      DO 33 J=1,6
   33 ES(J)=SAS(J) +DS(J)*CORCB +(BS(J)+FS(J))*GAM
      CNDP(2)=SC(1)*(4.*ES(1)+2.*ES(2))+2.*SC(2)*ES(1)
      DUM=0.0
      DO 34 J=2,5
   34 DUM=DUM+SC(J+1)*ES(J)-SC(J)*ES(J-1)
      CNDP(2)=(CNDP(2)+DUM)*PI*CD*SACA/2.
       CNDP(3)=C(1)*(4.*GS(1)+2.*GS(2))+2.*C(2)*GS(1)
      DUM=0.0
      DO 35 J=2,5
   35 DUM=DUM+C(J+1)*GS(J)-C(J)*GS(J+1)
      CNDP(3)=(CNDP(3)+DUM)*PI*CD*SACA/2.*GAM
      CNDP(4)=PI*CD*SACA*(2.*SC(1)-SC(2))
      CNDP(1)=CNDP(2)+CNDP(3)+CNDP(4)
C
C     COMPUTE DUCT MOMENT COEFFICIENTS
C
      CMDP(2)=C(1)*(4.*GS(1)+4.*GS(2)+2.*GS(3))+C(2)*(GS(2)-GS(4))
     1        +C(3)*(2.*GS(1)-GS(5))-C(4)*(GS(2)-GS(6))+C(5)*GS(3)
     2      +C(6)*GS(4)
      CMDP(2)=CMDP(2)*PI/4.*GAM*SACA*CD*CD
      CMDP(3)=PI/2.*CD*CD*SACA*(2.*SC(1)+SC(3))
      CMDP(4)=SC(1)*(4.*ES(1)+4.*ES(2)-2.*ES(3))+SC(2)*(ES(2)-ES(4))
     1     +SC(3)*(2.*ES(1)-ES(5))+SC(4)*(ES(2)-ES(6))
```

```
      2      +SC(5)*ES(3)+SC(6)*ES(4)
      CMDP(4)=CMDP(4)*PI/4.*SACA*CD*CD
      DO 36 J=1,6
36    E(J)=E(J)-F(J)*GAM
      CMDP(5)=SC(1)*(4.*E(1)+2.*E(2))+2.*SC(2)*E(1)
      DUM=0.0
      DO 37 J=2,5
37    DUM=DUM+SC(J+1)*E(J)-SC(J)*E(J+1)
      CMDP(5)=-(CMDP(5)+DUM)*PI/2.*SACA*CD
      DO 38 J=2,5
38    CMDP(1)=CMDP(1)+CMDP(J)
23    CONTINUE
      PRINT 153
      PRINT 154, (CTDP(N),N=1,5),CNDP(1),CMDP(1)
      CNDP(1)=CNDP(1)*CTCF
      CMDP(1)=CMDP(1)*CTCF*RRP/2.0
      PRINT 155, (CTDP(N),N=6,10),CNDP(1),CMDP(1)
       PRINT 156
      N=0
       NTOT=0
70     DO 71 J=1,NZ
      IF(STALL(J))71,71,72
72    N=N+1
      JSTL(N)=J
      NTOT=N
71    CONTINUE
73    IF(NTOT)75,75,74
74    PRINT 157,(JSTL(N),N=1,NTOT)
75    CONTINUE
      IF(NPHI)99,99,80
C
C    COMPUTD DUCT SURFACE PRESSURE COEFFICIENTS
C
80    CALL PRESS
      DO 81 I=1,NPHI
      DO 81 J=1,IR
      IF(XPRES(J)-XP)81,82,82
82    CPP(I,J)=CPP(I,J)+DELP*CSALF*CSALF
81    CONTINUE
      IF(NPRES.LE.1) GO TO 85
      DO 83 I=1,NPHI
      DO 83 J=1,IR
      CPP(I,J)=CPP(I,J)*ARJV*ARJV/2.0
83    CPM(I,J)=CPM(I,J)*ARJV*ARJV/2.0
85    NPAG=NPAG+1
      PRINT 703,NRUN,NPAG
      PRINT 102,TALK
      PRINT 148,CD,XP,TC,RRP,RCBRP,APA
      PRINT 149,ALF,ARJV,ARJVP,ARJ,ARJP
      PRINT 250,(PHI(J),J=1,NPHI)
      GO TO (1,2,3,4,5),NPHI
1     PRINT251
      GO TO 6
2      PRINT 252
```

```
      GO TO 6
3     PRINT 253
      GO TO 6
4      PRINT 254
      GO TO 6
5      PRINT 255
6     CONTINUE
      DO 86 J=1,IR
86    PRINT 245,XPRES(J),( CPP(I,J),CPM(I,J), I=1,NPHI )
      IF(NPRES.LE.1) GO TO 8
      PRINT 256
       GO TO 99
8      PRINT 257
99    RETURN
      END
```

```
$IBJOB          MAP
$IBFTC MAIN
      DIMENSION ALPHAR(48),HOLVEL(16)
      DIMENSION XV(48),DELTAX(48),XP(64),YV(48),S(48),YP(64)
      DIMENSION N1(10),L1(10)
       DIMENSION DWASH(48),C(6)
      COMMON/COM1/HOLVEL,NNN
      COMMON /COM2/ N,NN,M,XV,DELTAX,XP,YV,S,YP
      COMMON/COM3/CB,NF,N1,L1
      PI=ATAN(1.0)*4.0
C  N= NO. OF VORTICES,  M= NO. OF SPAN STATIPNS, NN= NO. OF CONTROL POINTS
C      IF N=NN  HOLE IN WING ABSENT, WING FORCES CALCULATED
      READ 501,N,NN,M
501   FORMAT(3I3)
      PRINT 502,N,M,NN
502    FORMAT(1X,*N=*,I5,*M =*,I5,*NN=*,I5)
      READ 100,(XV(I),DELTAX(I),XP(I),YV(I),S(I),YP(I),I=1,N)
100   FORMAT(6F10.6)
      PRINT 300,(I,XV(I),DELTAX(I),XP(I),YV(I),S(I),YP(I),I=1,N)
C     CBAR = ROOT CHORD
       CBAR=40.0
      CB=CBAR/5.0
      DO 203 I=1,N
       XV(I)=XV(I)/CB
      DELTAX(I)=DELTAX(I)/CB
      XP(I)=XP(I)/CB
      YV(I)=YV(I)/CB
      S(I)=S(I)/CB*0.5
203    YP(I)=YP(I)/CB
      PRINT 301
      PRINT 300,(I,XV(I),DELTAX(I),XP(I),YV(I),S(I),YP(I),I=1,N)
300   FORMAT(1X,I5,6F10.5)
301   FORMAT(1X,*I*,6X,*XV(I)   *,4X,*DELTAX*,4X,*XP*,8X,*YV*,8X,*2S*,9X,
     1*YP*//)
      IF(NN-N)20,21,20
20     NF=N+1
      READ101,(XP(I),YP(I),I=NF,NN )
101   FORMAT(2F10.5)
       PRINT 313
      PRINT312,(I,XP(I),YP(I),I=NF,NN)
      DO 204  I=NF,NN
      XP(I)=XP(I)/CB
      YP(I)=YP(I)/CB
204    CONTINUE
      PRINT 312,(I,XP(I),YP(I),I=NF,NN)
312   FORMAT(1X,I3,10X,E15.8,10X,E15.8)
313   FORMAT(1X,*I=*,12X,*XP(I)*    10X,*YP(I)*,*VALUES FOR HOLE*)
21    CONTINUE
C   IFNNN IS +VE ONLY VELOCITY IN THE HOLE CALCULATED
C   NNNIS 0  ONLY FORCES ARE CALCULATED
C     NNN IS -VE BOTH ARE CALCULATED
C N1=NO. OF VORTICES AT A GIVEN SPAN STATION  PLUS ONE
C    L1= NUMBER OF THE FIRST VORTEX AT THE SPAN STATION
C    K2= N3. OF THE LAST VORTEX AT THE GIVEN SPAN STATION
      READ 503,(N1(I),I=1,M)
```

```
503     FORMAT(10I5)
        PRINT 507
        PRINT 506,(I,N1(I),L1(I),        I=1,M)
506     FORMAT(3(4X,I5))
507     FORMAT(1X,*STATION*,2X,*N1*,7X,*L1*)
        A1=1.0
        DO 11 I=1,48
11        ALPHAR(I)=A1*PI/180.0
C     IF  NEXT  IS ZERO OR -VE THEN SUBROUTINE WING IS CALLED
C    IF NEXT IS +VE THEN SUBROUTINE EFFECT IS CALLED
        NEXT=10.0
        IF(NEXT)1,1,2
C
C               DATA
2       ALF=88.0*PI/180.0
        GAM=61.53817*COS(ALF)
        READ 102,(C(I),I=1,6)
102     FORMAT(6F10.5)
C
        CALL EFFECT(ALF,GAM,C,DWASH)
        DO 12 I=1,N
12        ALPHAR(I)=DWASH(I)+ALPHAR(I)
          PRINT 509
        PRINT 508,(ALPHAR(I),I=1,N)
509     FORMAT(1X,*   THE DOWN WASH  MATRIX*//)
508       FORMAT(10(2X,F10.6))
          NNN=-10
1         CONTINUE
        CALL WING(ALPHAR)
        STOP
        END
```

```
$IBFTC WING
      SUBROUTINE WING(ALPHA)
      DIMENSION XV(48),DELTAX(48),XP(64),YV(48),YP(64),S(48),F(48,48)
     1,AL1(48),AM1(48),B(48,1)
      DIMENSION ALPHA(48),HOLVEL(16),HF(16,48),GAMWG(48)
       DIMENSION CLL(11),CML(11),N1(10),L1(10)
       DIMENSIONCLX(12)
      COMMON/COM1/HOLVEL,NNN
      COMMON /COM2/ N,NN,M,XV,DELTAX,XP,YV,S,YP
      COMMON/COM3/CB,NF,N1,L1
       PI=ATAN(1.0)*4.0
      DO 200 I=1,NN
      DO 200 J=1,N
      X=XP(I)-XV(J)
      Y=YP(I)-YV(J)
      YBAR=YP(I)+YV(J)
      AB1=Y+S(J)
      AB2=Y-S(J)
      AB3=SQRT(X**2+(Y+S(J))**2)
      AB4=SQRT(X**2+(Y-S(J))**2)
      AB5=YBAR+S(J)
      AB6=YBAR-S(J)
      AB7=SQRT(X**2+AB5**2)
      AB8=SQRT(X**2+AB6**2)
      IF(X.EQ.0.0)F11=0.0
      IF(X.EQ.0.0)F13=0.0
      IF(X.EQ.0.0)GO TO 215
      F11=AB1/(X*AB3)
      F13=AB2/(X*AB4)
215    F12=1./AB1*(1.+X/AB3)
      F14=1./AB2*(1.-X/AB4)
      F1=F11+F12-F13-F14
      IF(X.EQ.0.0)F21=0.0
      IF(X.EQ.0.0)F23=0.0
      IF(X.EQ.0.0)GO TO 216
      F21=AB5/(X*AB7)
      F23=AB6/(X*AB8)
216   F22=1./AB5*(1.-X/AB7)
      F24=1./AB6*(1.+X/AB8)
      F2=F21+F22-F23-F24
      IF(I.GT.N) GO TO 202
      F(I,J) =F1+F2
      GO TO 200
202    II=I-N
       HF(II,J)=F1+F2
200    CONTINUE
       PRINT 510,(F(I,1),I=1,N)
510   FORMAT(1X,13F10.5)
      DO208 I=1,N
208   B(I,1)=0.0
       CALL MATINV(F,48,N,B,1,1)
      K=NN-N
      DO 401 I=1,N
      AL1(I)=0.0
      GAMWG(I)=0.0
       IF(I.GT.K) GO TO 401
```

```
      HOLVEL(I)=0.0
4^1   CONTINUE
      DO 402 I=1,N
      DO 400 J=1,N
      F(I,J)=8.0*PI*F(I,J)
      IF (NNN)1,2,1
1      GAMWG(I)=GAMWG(I)+F(I,J)*ALPHA(J)
5     IF(NNN)4,4,3
3      GO TO 400
4     CONTINUE
2      F(I,J)=F(I,J)/DELTAX(I)
      AL1(I)=AL1(I)+F(I,J)*ALPHA(J)
4^0   CONTINUE
      IF(NNN.GT.0) GO TO 402
       AXLE=YV(I)/6.^
4^2   CONTINUE
       AM1(I)=(XV(I)-AXLE)*AL1(I)
      IF(NNN.EQ.0) GO TO 14
       DO 403 I=1,K
       DO 403 J=1,N
       HOLVEL(I)=HOLVEL(I)+1.0/8.0/PI*HF(I,J)*GAMWG(J)
403   CONTINUE
      PRINT 509
       PRINT508,(HOLVEL(I),I=1,K)
508   FORMAT(1X,8(1X,E15.8))
5^9   FORMAT (1X,*VELOCITIES IN THE HOLE DUE TO THE WING*)
      IF(NNN.GT.0)  GO TO 999
14     CONTINUE
      PRINT 307
        PRINT 306,(AL1(I),I=1,N)
307   FORMAT(1X,*      LMATRIX)*)
306    FORMAT(8(1X,E15.8))
      PRINT 308
      PRINT 306,(AM1(I),I=1,N)
308   FORMAT(1X,*          M  MATRIX*)
C    TO GET CLL (LOCAL LIFT COEFF)
      DO 888 IJ=1,M
      CALL VARSIM(N1(IJ),XV,CLL(IJ),AL1,L1(IJ),^,XP,N,N,NN)
      CALL CURFIT(XV,YV,L1(IJ),AL1,AA1)
13     CLL(IJ)=CLL(IJ)+AA1
888    CONTINUE
      PRINT 310
      PRINT 309,(CLL(I),I=1,M)
309   FORMAT(1X,8E15.7)
310   FORMAT(1X,*LOCAL LIFT COEFF  *)
C     TO GET CML (LOCAL MOMENT COEFF)
       DO 889 IJ=1,M
      CALL VARSIM(N1(IJ),XV,CML(IJ),AM1,L1(IJ),0,XP,N,N,NN)
      IL=L1(IJ)
      AXLE=YV(IL)/6.0
      AA1=0.5*(XV(IL)-AXLE)*AM1(IL)
      CML(IJ)=CML(IJ)+AA1
```

```
889    CONTINUE
       PRINT 311
311     FORMAT(1X,*  LOCAL  MOMENT COEFF*)
       PRINT 309,(CML(I),I=1,M)
C     TO GET    CL  AND  CM
       NA=M+1
        DO 220 I=1,NA
        IF(I.EQ.NA) GO TO 221
        K3=L1(I)
       CLX(I)=YV(K3)
       GO TO 220
221     CLX(I)=60.0/(CB*5.0)
        CLL(NA)=0.0
       CML(NA)=0.0
220     CONTINUE
        Q=(CLL(2)-CLL(1))/(CLX(2)-CLX(1))
        Q1=(CML(2)-CML(1))/(CLX(2)-CLX(1))
        CALL VARSIM(NA,CLX,CL,CLL,1,1,XP,12,11,NN)
       CALL VARSIM(NA,CLX,CM,CML,1,1,XP,12,11,NN)
       CL=CL+0.5*(Q*CLX(1)+CLL(1))*CLX(1)
       CM=CM+0.5*(Q1*CLX(1)+CML(1))*CLX(1)
       PRINT 222,CL,CM
222    FORMAT(1X,*  CL=*,E15.7,*     CM=*,E15.7)
999     RETURN
       END
```

```
$IBFTC INVERS
      SUBROUTINE MATINV(A,NN,N,B,MM,M)
      DIMENSION A(NN,NN),B(NN,MM),PIVOT(48),IPIVOT(48),INDEX(48,2)~~L3 3P 134 L7 P737~~
C     INITIALIZATION
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C     SEARCH FOR PIVOT ELEMENT
40    AMAX =0.0
   45 DO 105 J=1,N
   50 IF(IPIVOT(J)-1)60,105,60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1) 80, 100, 740
80     IF(ABS(AMAX)-ABS(A(J,K)))85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON D+AGONA-
  130  IF(IROW-ICOLUM) 140,260,140
  140  CONTINUE
  150 DO 200 L=1,N
  160  SWAP=A(IROW,L)
 170  A(IROW,L)= A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205 IF(M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250  B(ICOLUM,L)=SWAP
 260   INDEX(I,1)= IROW
 270   INDEX(I,2)= ICOLUM
  310  PIVOT(I)=A(ICOLUM,ICOLUM)
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
330   A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C     REDUCE NON-PIVOT ROWS
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
420    A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C     INTERCHANGE COLUMNS
  600 DO 710 I=1,N
```

```
  610 L=N+1-I                                        F4020074
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630       F4020075
  630 JROW=INDEX(L,1)                                F4020076
  640 JCOLUM=INDEX(L,2)                              F4020077
  650 DO 705 K=1,N                                   F4020078
  660 SWAP=A(K,JROW)                                 F4020079
  670 A(K,JROW)=A(K,JCOLUM)                          F4020030
  700 A(K,JCOLUM)=SWAP                               F4020031
  705 CONTINUE                                       F4020032
710   CONTINUE
740   RETURN
       END
```

```
$IBFTC VARSIM
      SUBROUTINE VARSIM(N,X,SUM,FU,L1,J1,XP,I1,I2,I3)
C  IF J1=0 THEN THE LAST VALUE IN THE LAST INTERVAL =0AND X =XP(L2+1)
C  N  SHOULD BE ODD SO THAT THE NO. OF INTERVALS ARE EVEN
      DIMENSION X(I1),FU(I2),XP(I3)
       SUM=0.0
       N1=(N-1)/2
      H=X(L1+1)-X(L1)
       H=ABS(H)
      DO 100 II=1,N1
      L=2*II-1
      L2=L1+L-1
      IF(J1.GT.0)GO TO 300
      IF(II.EQ.N1)AK1=XP(L2+1)
      IF(II.EQ.N1)AK2=0.0
      IF(II.EQ.N1) GO TO 301
300   CONTINUE
      AK1=X(L2+2)
      AK2=FU(L2+2)
301   ALP1=(X(L2+1)-X(L2))/H
       ALP2=(AK1-X(L2+1))/H
      ALP1=ABS(ALP1)
      ALP2=ABS(ALP2)
      S1=FU(L2)*ALP2*(ALP1**2+0.5*ALP1*ALP2-0.5*ALP2**2)
      S2=0.5*FU(L2+1)*(ALP1+ALP2)**3
      S3=AK2*(ALP1**3-ALP1**2*ALP2-ALP2**2*ALP1+1.5*ALP1*ALP2-1.5*ALP1
     1**2)
      S4=H/(3.0*ALP1*ALP2)*(S1+S2+S3)
      SUM=SUM+S4
100    CONTINUE
      RETURN
      END
```

```
$IBFTC CUR
      SUBROUTINE CURFIT(X,YV,L1,AL,AREA)
C     USING CURVE A *X**(-0.5)+B*X**_.5+C*X**1. WHERE X=0 IS THE L.E
      DIMENSION AEF(3,3),X(48),AEB(3,1),AL(48) ,YV(48)
      XLE=YV(L1)/(2.*3.)
        DO 100 I=1,3
       K=L1+I-1
      DO 101 J=1,3
       AEF(I,J)=(X(K)-XLE)**(FLOAT(J)-1.5)
101    CONTINUE
        AEB(I,1)=AL(K)
100    CONTINUE
      CALL MATINV(AEF,3   ,3,AEB,1,1)
      XLE=X(L1)_XLE
       AREA=AEB(1,1)*XLE**0.5/0.5+AEB(2,1)*XLE**1.5/1.5+AEB(3,1)*XLE
     1**2.5/2.5
      RETURN
      END
```

```
$IBFTC EFFECT
      SUBROUTINE EFFECT(ALPHA,GAM,C,DWASH)
      DIMENSION XXP(21),Y(21),PHIP(21),V2(21),E(21,21),C(6)
      DIMENSION DWASH(48)
      DIMENSION XV(48),DELTAX(48),XP(64),YV(48),S(48),YP(64)
      COMMON /COM2/ N,NN,M,XV,DELTAX,XP,YV,S,YP
      COMMON MZZZ
       DUM=0.0
      MZZZ=0
       CALL ELLIPS(DUM,DUM,DUM)
      PI=4.0*ATAN(1.0)
C
C               DATA
      CD=0.1999
       XF=20.0/40.0
      YF=26.5/40.0
      CROOT=40.0
      RTE=20.125/2.0
C
C
      PRINT 105,(C(I),I=1,6)
105   FORMAT(1X,*C(I)=  *,6E15.7)
      DO 999 JK=1,N
      RR=SQRT((XP(JK)-XF)**2+(YP(JK)-YF)**2)
      RR=RR*CROOT/RTE
      PHI=ABS((XP(JK)-XF)/(YP(JK)-YF))
       PHI=ATAN(PHI)
      IF(XP(JK)-XF)31,40,33
33    IF(YP(JK)-YF)37,40,39
37    PHI=1.5*PI-PHI
      GO TO 40
39    PHI=PI*0.5+PHI
      GO TO 40
31    IF(YP(JK)-YF) 34,40,36
36    PHI=PI*0.5-PHI
      GO TO 40
34     PHI=1.5*PI+PHI
       GO TO 40
40      CONTINUE
       X=0.0
       NK=21
      C1=CD*2.0
C     PRINT 301,JK,RR,PHI
C 301   FORMAT(1X,*  JK=*,I5,*     RR=*,E15.7,*        PHI=*,E15.7)
      STEP=C1/FLOAT(NK-1)
       XXP(1)=C1/2.0
      STEP2=2.0*PI/FLOAT(NK-1)
      PHIP(1)=0.0
      DO 100 I=1,NK
      DO 100 J=1,NK
      IF(J.EQ.1) GO TO 200
      PHIP(J)=PHIP(J-1)+STEP2
200   CONTINUE
      IF(J.GT.1) GO TO 201
        IA=I-1
      IF(I.EQ.1) GO TO 101
       XXP(I)=XXP(IA)-STEP
101    CONTINUE
      T1=2.0*XXP(I)/C1
       THETA=ARCOS(T1)
```

```
      XXP(I)=XXP(IA)-STEP
101   CONTINUE
      T1=2.0*XXP(I)/C1
      THETA=ARCOS(T1)
      GAMD=C(1)*TAN(PI/2.0-THETA/2.0)+C(2)*SIN(THETA)+C(3)*SIN(2.0*THETA
     1)+C(4)*SIN(3.0*THETA)+C(5)*SIN(4.0*THETA)+C(6)*SIN(5.0*THETA)
      IF(ABS(T1).GT.0.999) GAMD=0.0
      S1=GAMD/(PI*2.0*SQRT((X-XXP(I))**2+(RR+1.0)**2))
      AKP=SQRT(4.0*RR/((X-XXP(I))**2+(RR+1.0)**2))
      MZZZ=1
      CALL ELLIPS(AKP,TK,TE)
      Y(I)=S1*(TK-(1.0+2.0*(RR-1.0)/((X-XXP(I))**2+(RR-1.0)**2))*TE )
201   CONTINUE
      E1=COS(PHIP(J))*(RR*COS(PHI-PHIP(J))-1.0)
      E2=(X-XXP(I))**2+RR**2+1.0-2.0*RR*COS(PHI-PHIP(J))
      E(I,J)=GAMD*E1/E2**1.5
100   CONTINUE
C     PRINT 509,(Y(I),I=1,NK)
C509   FORMAT(1X,*Y(I)--->*,8E15.7)
C     PRINT 505,(XXP(I),I=1,NK)
C505  FORMAT(1X,*XXP ARE   --*,11F10.5)
      CALL VARSIM(NK,XXP,UGMD,Y,1,1,XP,NK,NK,NN)
C     PRINT 500,UGMD,GAM,GAMD
C500   FORMAT(1X,*  UGMD=*,E15.7,*     GAM=*,E15.7,*     GAMD=*,E15.7)
      UGMD=UGMD*GAM
      DO 202 I=1,NK
      DO 203 J=1,NK
      Y(J)=E(I,J)
203   CONTINUE
      CALL VARSIM(NK,PHIP,V1,Y,1,1,XP,NK,NK,NN)
      V2(I)=V1
202   CONTINUE
      CALL  VARSIM(NK,XXP,UGALF,V2,1,1,XP,NK,NK,NN)
      UGALF=-UGALF/(4.0*PI)*ATAN(ALPHA)
      PRINT 302,UGMD,UGALF
302   FORMAT(1X,*   UGMD=*,E15.7,*     UGALF=*,E15.7)
      DWASH(JK)=UGMD+UGALF
      DWASH(JK)=-DWASH(JK)
999   CONTINUE
      RETURN
      END
```

```
$IBFTC 03
      SUBROUTINE ELLIPS (AKSQ,TK,TE)
C
C     SUB. ELLIPS --TABLE LOOK UP OG ELLIPTIC INTEGERALS
C
      DIMENSION CKK(100),CK(100),CE(100)
      COMMON MZZZ
      IF (MZZZ) 73,10,3
10    CONTINUE
C
C           CKK= ARGUMENT OF ELLIPTIC INTEGRALS
C
      CKK (1 )=0.00
      CKK (2 )=0.01
      CKK (3 )=0.02
      CKK (4 )=0.03
      CKK (5 )=0.04
      CKK (6 )=0.05
      CKK (7 )=0.06
      CKK (8 )=0.07
      CKK (9 )=0.08
      CKK (1 0) =0.09
      CKK (1 1) =0.10
      CKK (1 2) =0.11
      CKK (1 3) =0.12
      CKK (1 4) =0.13
      CKK (1 5) =0.14
      CKK (1 6) =0.15
      CKK (1 7) =0.16
      CKK (1 8) =0.17
      CKK (1 9) =0.18
      CKK (2 0) =0.19
      CKK (2 1) =0.20
      CKK (2 2) =0.21
      CKK (2 3) =0.22
      CKK (2 4) =0.23
      CKK (2 5) =0.24
      CKK (2 6) =0.25
      CKK (2 7) =0.26
      CKK (2 8) =0.27
      CKK (2 9) =0.28
      CKK (3 0) =0.29
      CKK (3 1) =0.30
      CKK (3 2) =0.31
      CKK (3 3) =0.32
      CKK (3 4) =0.33
      CKK (3 5) =0.34
      CKK (3 6) =0.35
      CKK (3 7) =0.36
      CKK (3 8) =0.37
      CKK (3 9) =0.38
       CKK(40 )=0.39
      CKK (4 1) =0.40
      CKK (4 2) =0.41
      CKK (4 3) =0.42
```

```
CKK(44)=0.43
CKK(45)=0.44
CKK(46)=0.45
CKK(47)=0.46
CKK(48)=0.47
CKK(49)=0.48
CKK(50)=0.49
CKK(51)=0.50
CKK(52)=0.51
CKK(53)=0.52
CKK(54)=0.53
CKK(55)=0.54
CKK(56)=0.55
CKK(57)=0.56
CKK(58)=0.57
CKK(59)=0.58
CKK(60)=0.59
CKK(61)=0.60
CKK(62)=0.61
CKK(63)=0.62
CKK(64)=0.63
CKK(65)=0.64
CKK(66)=0.65
CKK(67)=0.66
CKK(68)=0.67
CKK(69)=0.68
 CKK(70)=0.69
CKK(71)=0.70
CKK(72)=0.71
CKK(73)=0.72
CKK(74)=0.73
CKK(75)=0.74
CKK(76)=0.75
CKK(77)=0.76
CKK(78)=0.77
CKK(79)=0.78
CKK(80)=0.79
CKK(81)=0.80
CKK(82)=0.81
CKK(83)=0.82
CKK(84)=0.83
CKK(85)=0.84
CKK(86)=0.85
CKK(87)=0.86
CKK(88)=0.87
CKK(89)=0.88
CKK(90)=0.89
 CKK(91)=0.90
CKK(92)=0.91
CKK(93)=0.92
CKK(94)=0.93
CKK(95)=0.94
CKK(96)=0.95
CKK(97)=0.96
CKK(98)=0.97
```

```
CK(52)=1.862641
CK(53)=1.87140^
CK(54)=1.880361
CK(55)=1.889533
CK(56)=1.898925
CK(57)=1.908547
CK(58)=1.918410
CK(59)=1.928526
CK(60)=1.938908
CK(61)=1.949568
CK(62)=1.960521
CK(63)=1.971783
CK(64)=1.983371
CK(65)=1.995303
CK(66)=2.^07598
CK(67)=2.020279
CK(68)=2.033369
CK(69)=2.046894
CK(70)=2.060882
CK(71)=2.075363
CK(72)=2.090373
CK(73)=2.105948
CK(74)=2.122132
CK(75)=2.138970
CK(76)=2.156516
CK(77)=2.174827
CK(78)=2.193971
CK(79)=2.214022
CK(80)=2.235068
CK(81)=2.257205
CK(82)=2.280549
CK(83)=2.305232
CK(84)=2.331409
CK(85)=2.359264
CK(86)=2.389016
CK(87)=2.420933
CK(88)=2.455338
CK(89)=2.492635
CK(90)=2.533335
CK(91)=2.578092
CK(92)=2.627773
CK(93)=2.683551
CK(94)=2.747073
CK(95)=2.820752
CK(96)=2.908337
CK(97)=3.^16112
CK(98)=3.155875
CK(99)=3.354141
CK(100)=3.695637

    CE=COMPLETE ELLIPTIC INTEGERAL OF SECOND KIND

CE(1)=1.570796
CE(2)=1.566862
CE(3)=1.562913
```

CE(4)=1.558948
CE(5)=1.554969
CE(6)=1.550973
CE(7)=1.546963
CE(8)=1.542936
CE(9)=1.538893
CE(10)=1.534833
CE(11)=1.530758
CE(12)=1.526665
CE(13)=1.5225550
CE(14)=1.518428
CE(15)=1.514284
CE(16)=1.510122
CF(17)=1.505942
CE(18)=1.501743
CE(19)=1.497526
CE(20)=1.493290
CE(21)=1.489035
CE(22)=1.4847610
CE(23)=1.480466
CE(24)=1.476152
CE(25)=1.471818
CE(26)=1.467462
CE(27)=1.463086
CE(28)=1.458688
CE(29)=1.454269
CE(30)=1.449827
CE(31)=1.445363
CE(32)=1.440876
CE(33)=1.436366
CE(34)=1.431832
CE(35)=1.477274
CE(36)=1.422691
CE(37)=1.418083
CE(38)=1.413450
CE(39)=1.408791
CE(40)=1.404105
CE(41)=1.399392
CE(42)=1.394652
CE(43)=1.389883
CE(44)=1.385086
CE(45)=1.380259
CE(46)=1.375402
CE(47)=1.370515
CE(48)=1.365596
CE(49)=1.360645
CE(50)=1.355661
CE(51)=1.350644
CE(52)=1.345592
CE(53)=1.340505
CE(54)=1.335382
CE(55)=1.330223
CE(56)=1.325024
CE(57)=1.319788
CE(58)=1.314511

```
      CE(59)=1.309192
      CE(60)=1.303832
      CE(61)=1.298428
      CE(62)=1.292979
      CE(63)=1.287484
      CE(64)=1.281942
      CE(65)=1.27635^
      CE(66)=1.270707
      CE(67)=1.265013
      CE(68)=1.259263
      CE(69)=1.253458
      CE(70)=1.247595
      CE(71)=1.241671
      CE(72)=1.235684
      CE(73)=1.229632
      CE(74)=1.223512
      CE(75)=1.217321
      CE(76)=1.211056
      CE(77)=1.204714
       CE(78)=1.198290
      CE(79)=1.191781
      CE(80)=1.185183
      CE(81)=1.178490
      CE(82)=1.171697
      CE(83)=1.164798
      CE(84)=1.157787
      CE(85)=1.150656
      CE(86)=1.143396
      CE(87)=1.135998
      CE(88)=1.128451
      CE(89)=1.1SS742
      CE(90)=1.112856
      CE(91)=1.104775
       CE(92)=1.096478
      CE(93)=1.^87938
      CE(94)=1.079121
      CE(95)=1.069986
      CE(96)=1.060474
      CE(97)=1.050502
      CE(98)=1.^39947
      CE(99)=1.^28595
      CE(100)=1.015994
C
      GO TO 30
  3   IF (AKSQ-.99)20,20,21
21    PARA=0.25*(1.0-AKSQ)
  700 TEST =1.00E-07
      IF(PARA-TEST)701,702,702
701   PARA =TEST
702   ZLP=ALOG(4./PARA )
      TK =ZLP*0.5*(1.+PARA)-PARA
      TE=1.0+(ZLP*PARA)-PARA
      GO TO 30
20    JA=100.0*AKSQ
      JA=1+JA
```

```
      IF(CKK(JA)-AKSQ)22,23,22
 23   TK=CK(JA)
      TE=CE(JA)
       GO TO 30
 22   CON=(AKSQ-CKK(JA))/(CKK(JA+1)-CKK(JA))
       TK=CK(JA)+CON*(CK(JA+1)-CK(JA))
      TE=CE(JA)^CON*(CE(JA+1)-CE(JA))
      GO TO 30
 73   IF(AKSQ-.^1)721,721,720
721   PARA=.25*ASSQ
      GO TO 700
720   AKSQ=1.-AKZZ
      GO TO20
 30   CONTINUE
      RETURN
      END
```

FAN OF NASA TN-D88

.199	0.8	0.06	1.006	0.108	0.01		
.0	0.0	0.0	0.0				
.04	0.78	0.005	0.8				
4	10	13	1	1	1		
.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75
.8	0.85	0.9	0.95	1.0			
.339	0.335	0.33	0.325	0.321	0.317	0.312	0.307
.3035	0.299	0.294	0.289	0.285			
41.8	38.5	37.8	36.5	35.0	33.8	32.5	31.2
30.0	29.0	27.5	26.5	25.5			
.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
.09	0.09	0.09	0.09	0.09			
[illegible]							
1	0	0.54	88.0				

Table 1: Input Data to Part I, Fan.

64 64 10

H4	0.83	0.94	0.615	1.23	0.615
1.48	1.08	2.02	0.615	1.23	0.615
2.625	1.25	3.27	0.615	1.23	0.615
3.96	1.43	4.70	0.615	1.23	0.615
5.43	1.43	6.13	0.615	1.23	0.615
7.01	1.80	7.93	0.615	1.23	0.615
.54	0.68	0.98	1.78	1.10	1.78
1.46	0.97	1.95	1.78	1.10	1.78
2.52	1.07	3.02	1.78	1.10	1.78
3.69	1.36	4.38	1.78	1.10	1.78
5.12	1.60	5.98	1.78	1.1	1.78
6.78	1.76	7.74	1.78	1.1	1.78
.65	0.6	1.06	2.82	1.0	2.82
1.53	1.03	2.09	2.82	1.0	2.82
2.58	1.04	3.13	2.82	1.0	2.82
3.7	1.25	4.38	2.82	1.0	2.82
5.1	1.44	5.82	2.82	1.0	2.82
6.62	1.72	7.54	2.82	1.0	2.82
.75	0.4	1.04	3.8	1.0	3.8
1.32	0.63	1.67	3.8	1.0	3.8
1.97	0.65	2.32	3.8	1.0	3.8
2.64	0.70	3.02	3.8	1.0	3.8
3.4	0.90	3.92	3.8	1.0	3.8
4.33	0.90	4.82	3.8	1.0	3.8
5.32	1.16	5.98	3.8	1.0	3.8
6.61	1.38	7.36	3.8	1.0	3.8
.94	0.25	1.06	4.8	1.0	4.8
1.30	0.48	1.54	4.8	1.0	4.8
1.74	0.36	1.90	4.80	1.0	4.80
2.66	0.84	3.02	4.8	1.0	4.8
3.42	0.86	3.88	4.8	1.0	4.8
4.31	0.92	4.8	4.8	1.0	4.8
5.36	1.1	5.9	4.8	1.0	4.8
6.53	1.3	7.2	4.8	1.0	4.8
1H04	0.21	1.17	5.8	1.0	5.8
1.33	0.39	1.56	5.8	1.0	5.8
1.8	0.35	1.90	5.8	1.0	5.8
2.64	0.83	3.03	5.8	1.0	5.8
3.40	0.85	3.88	5.8	1.0	5.8
4.30	0.92	4.8	5.8	1.0	5.8
5.28	1.05	5.85	5.8	1.0	5.8
6.39	1.18	7.03	5.8	1.0	5.8
1.24	0.27	1.4	6.8	1.0	6.8
1.61	0.43	1.83	6.8	1.0	6.8
2.09	0.53	2.36	6.8	1.0	6.8
2.62	0.64	3.0	6.8	1.0	6.8
3.42	0.85	3.85	6.8	1.0	6.8
4.33	0.95	4.8	6.8	1.0	6.8
5.27	0.92	5.72	6.8	1.0	6.8
6.3	1.18	6.9	6.8	1.0	6.8

Table 2: Input Data to Part II, Wing.

1.43	0.33	1.65	7.86	1.2	7.86
1.92	0.67	2.32	7.86	1.2	7.86
2.71	0.88	3.2	7.86	1.2	7.86
3.66	1.05	4.25	7.86	1.2	7.86
4.78	1.05	5.3	7.86	1.2	7.86
5.78	1.4	6.7	7.86	1.2	7.86
1.66	0.32	1.85	9.17	1.4	9.17
2.28	1.15	3.0	9.17	1.4	9.17
3.76	1.55	4.55	9.17	1.4	9.17
5.44	1.95	6.5	9.17	1.4	9.17
1.98	0.38	2.2	10.9	2.08	10.9
2.58	0.95	3.15	10.9	2.08	10.9
3.76	1.35	4.5	10.9	2.08	10.9
5.26	1.68	6.18	10.9	2.08	10.9

7	7	7	9	9	9	9	7	5	5
1	7	13	19	27	35	43	51	57	61

48 64 10

.4	0.83	0.94	0.615	1.23	0.615
1.48	1.08	2.02	0.615	1.23	0.615
2.625	1.25	3.27	0.615	1.23	0.615
3.96	1.43	4.70	0.615	1.23	0.615
5.43	1.43	6.13	0.615	1.23	0.615
7.01	1.80	7.93	0.615	1.23	0.615
.54	0.68	0.98	1.78	1.10	1.78
1.46	0.97	1.95	1.78	1.10	1.78
2.52	1.07	3.02	1.78	1.10	1.78
3.69	1.36	4.38	1.78	1.10	1.78
5.12	1.60	5.98	1.78	1.1	1.78
6.78	1.76	7.74	1.78	1.1	1.78
.65	0.6	1.06	2.82	1.0	2.82
1.53	1.03	2.09	2.82	1.0	2.82
2.58	1.04	3.13	2.82	1.0	2.82
3.7	1.25	4.38	2.82	1.0	2.82
5.1	1.44	5.82	2.82	1.0	2.82
6.62	1.72	7.54	2.82	1.0	2.82
.75	0.4	1.04	3.8	1.0	3.8
1.32	0.63	1.67	3.8	1.0	3.8
1.97	0.65	2.32	3.8	1.0	3.8
6.61	5.040	7.36	3.8	1.0	3.8
.94	0.25	1.06	4.8	1.0	4.8
1.30	0.48	1.54	4.8	1.0	4.8
1.74	0.36	1.90	4.80	1.0	4.80
6.53	5.02	7.2	4.8	1.0	4.8
1.04	0.21	1.17	5.8	1.0	5.8
1.33	0.39	1.56	5.8	1.0	5.8
1.8	0.35	1.90	5.8	1.0	5.8
6.39	4.83	7.03	5.8	1.0	5.8
1.24	0.27	1.4	6.8	1.0	6.8
1.61	0.43	1.83	6.8	1.0	6.8
2.09	0.53	2.36	6.8	1.0	6.8
6.3	4.54	6.9	6.8	1.0	6.8
1.43	0.33	1.65	7.86	1.2	7.86
1.92	0.67	2.32	7.86	1.2	7.86
2.71	0.88	3.2	7.86	1.2	7.86
3.66	1.05	4.25	7.86	1.2	7.36
4.78	1.05	5.3	7.86	1.2	7.86
5.78	1.4	6.7	7.86	1.2	7.86
1.66	0.32	1.85	9.17	1.4	9.17
2.28	1.15	3.0	9.17	1.4	9.17
3.76	1.55	4.55	9.17	1.4	9.17
5.44	1.95	6.5	9.17	1.4	9.17
1.98	0.38	2.2	10.9	2.08	10.9
2.58	0.95	3.15	10.9	2.08	10.9
3.76	1.35	4.5	10.9	2.08	10.9
5.26	1.68	6.18	10.9	2.08	10.9

Table 3: Input Data to Part 2, Wing with a duct.

3.02	3.8
3.92	3.8
4.5	3.8
5.0	3.8
3.02	4.8
3.88	4.8
4.8	4.8
5.9	4.8
3.03	5.8
3.88	5.8
4.8	5.8
5.85	5.8
3.0	6.8
3.85	6.8
4.5	6.8
5.0	6.8

7	7	7	5	5	5	5	7	5	5
1	7	13	19	23	27	31	35	41	45

.7159675 0.6394853-0.2924214 0.1800150-0.1350915 0.1013527

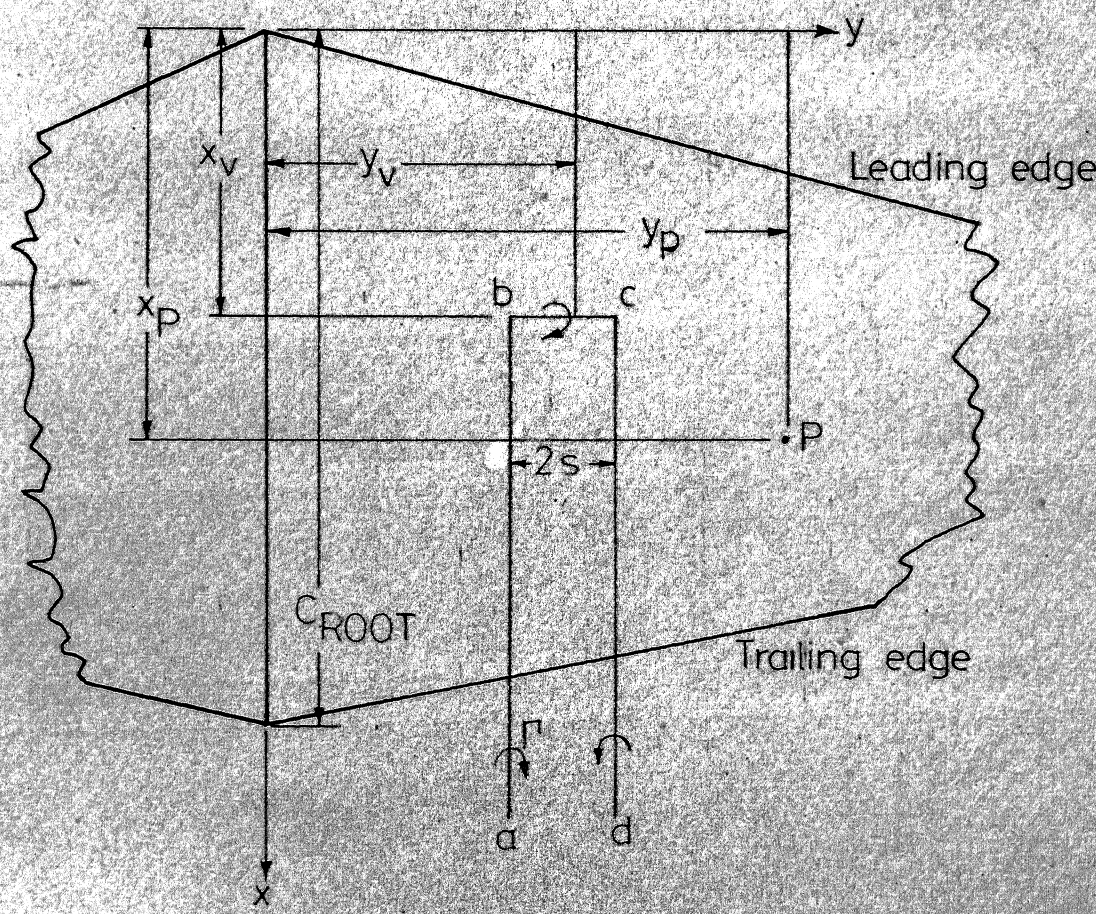


FIG. 1 _ A TYPICAL HORSE SHOE VORTEX AND CONTROL POINT

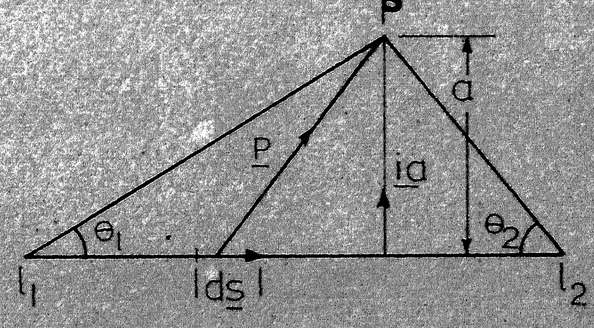


FIG. 2 _ A TYPICAL VORTEX ELEMENT AND CONTROL POINT

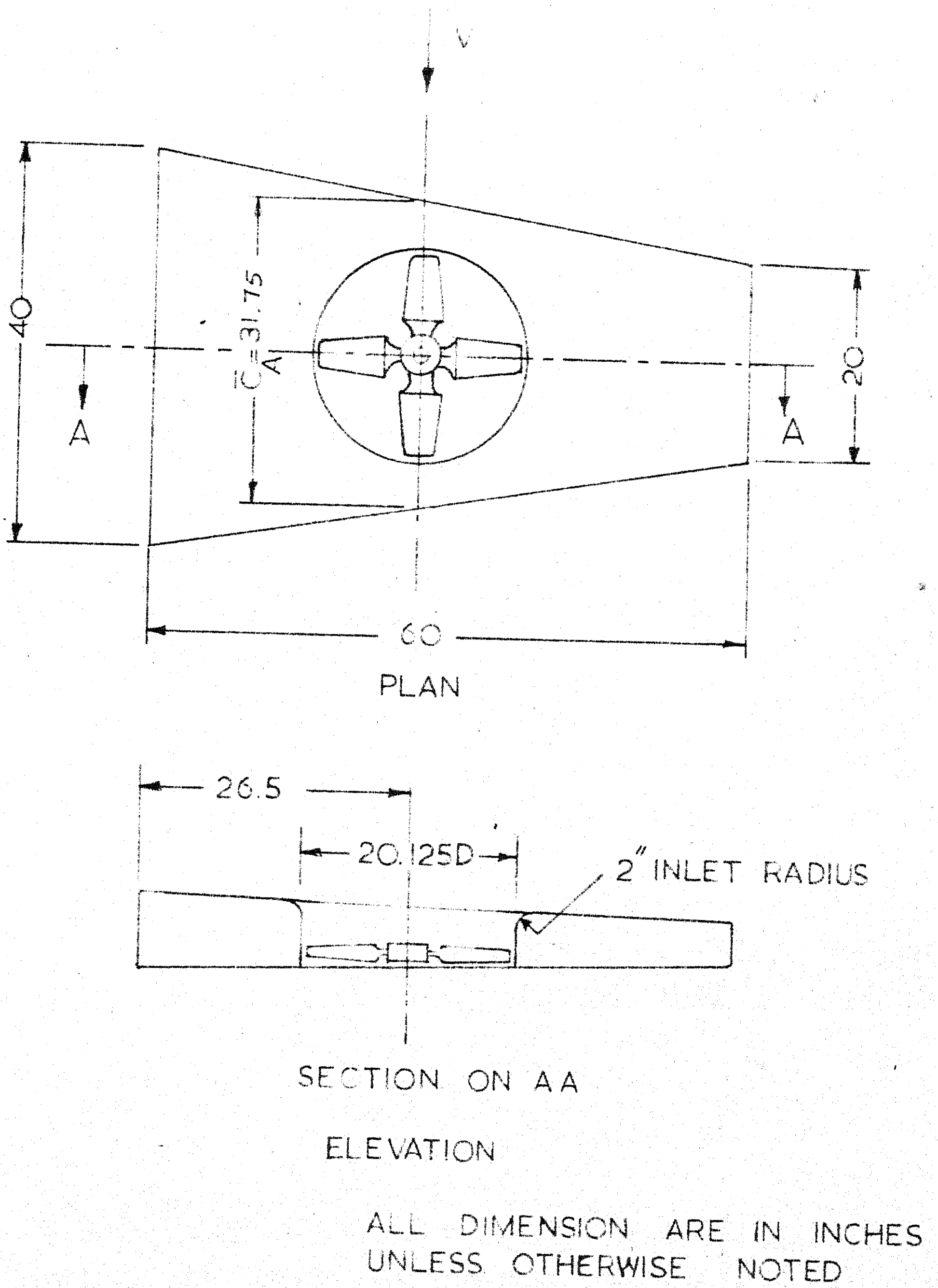


FIG. 3_GEOMETRIC CHARACHTERISTICS OF THE MODEL

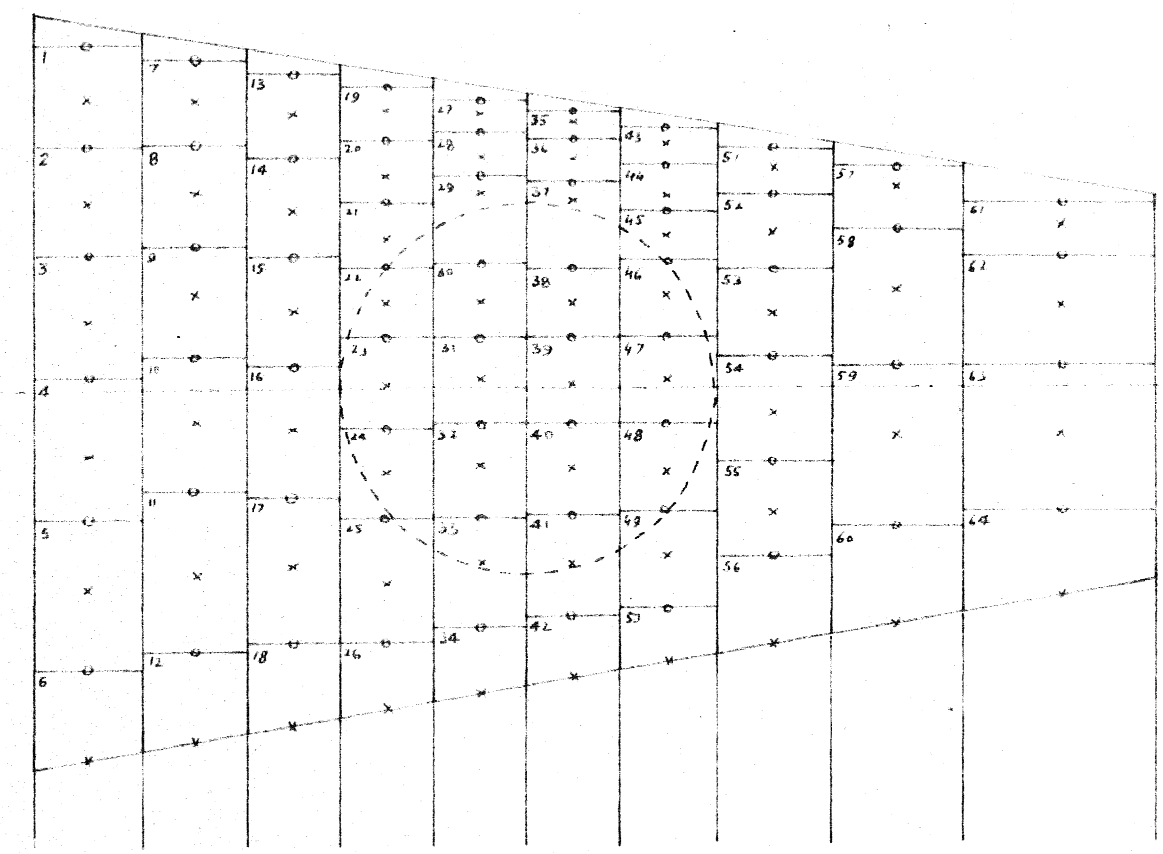


FIG. 4_ THE VORTEX AND CONTROL POINT DISTRIBUTION
FOR THE WING MODEL . n=64

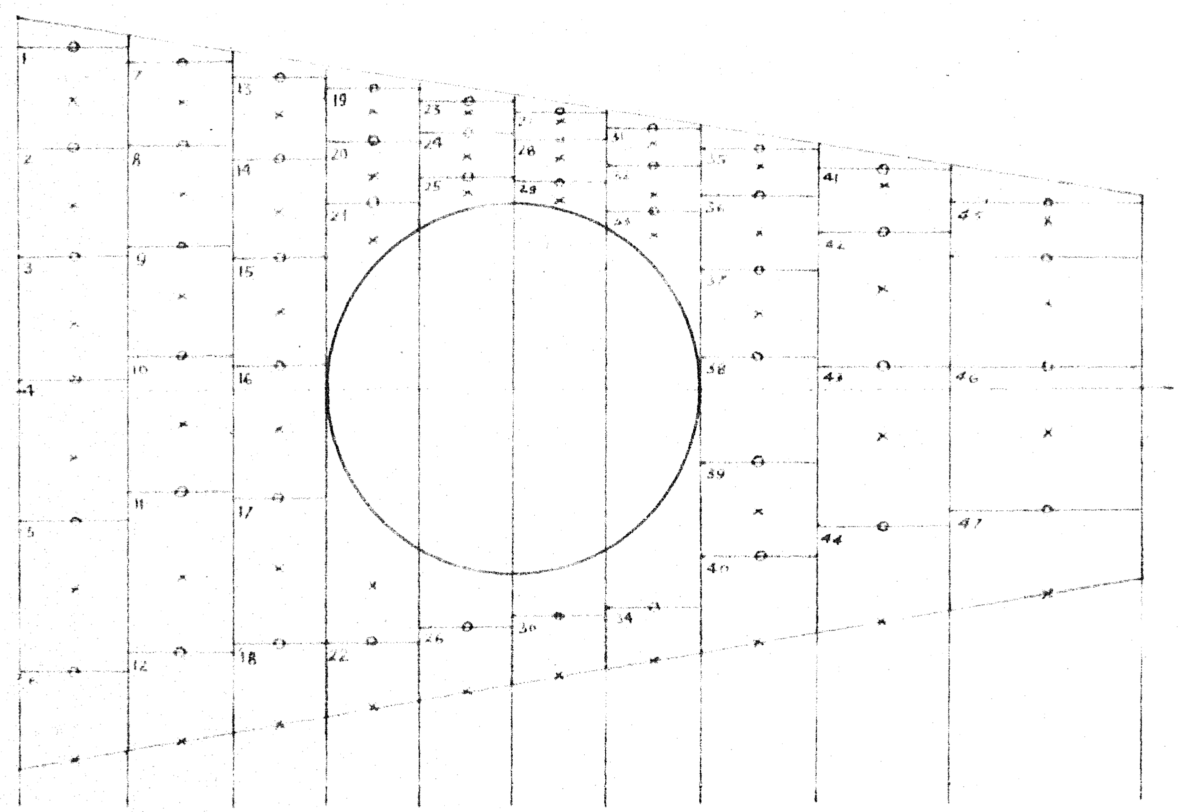


FIG. 5. THE VORTEX AND CONTROL POINT DISTRIBUTION FOR THE WING WITH A DUCT $n = 48$

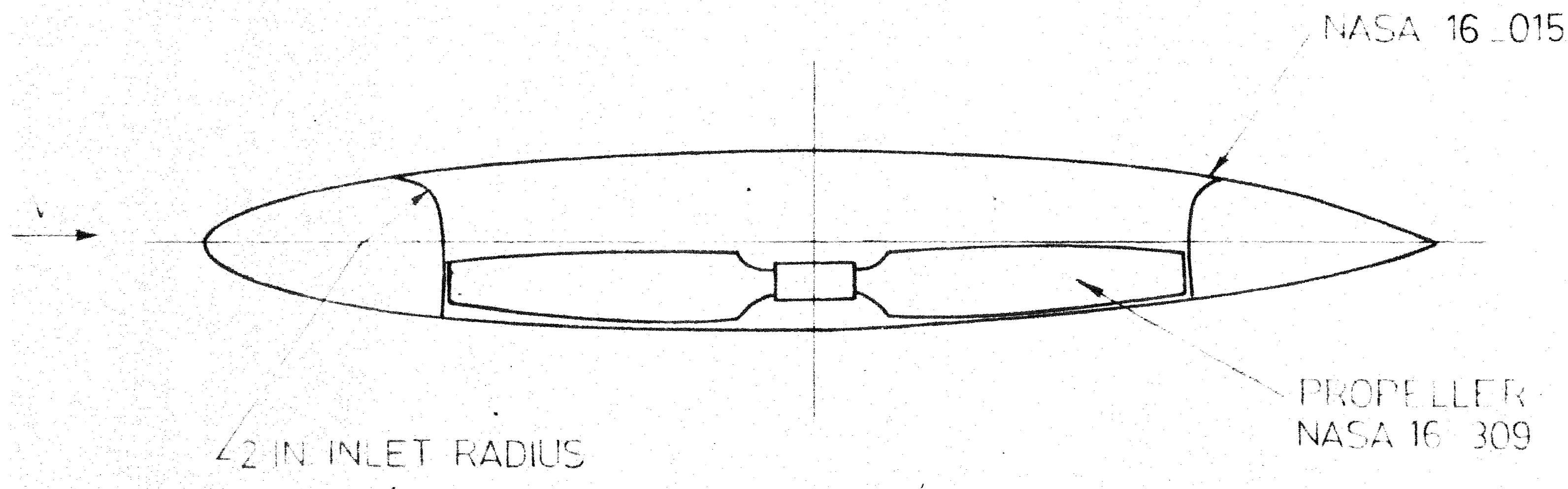


FIG 6 _ DETAIL OF THE FAN

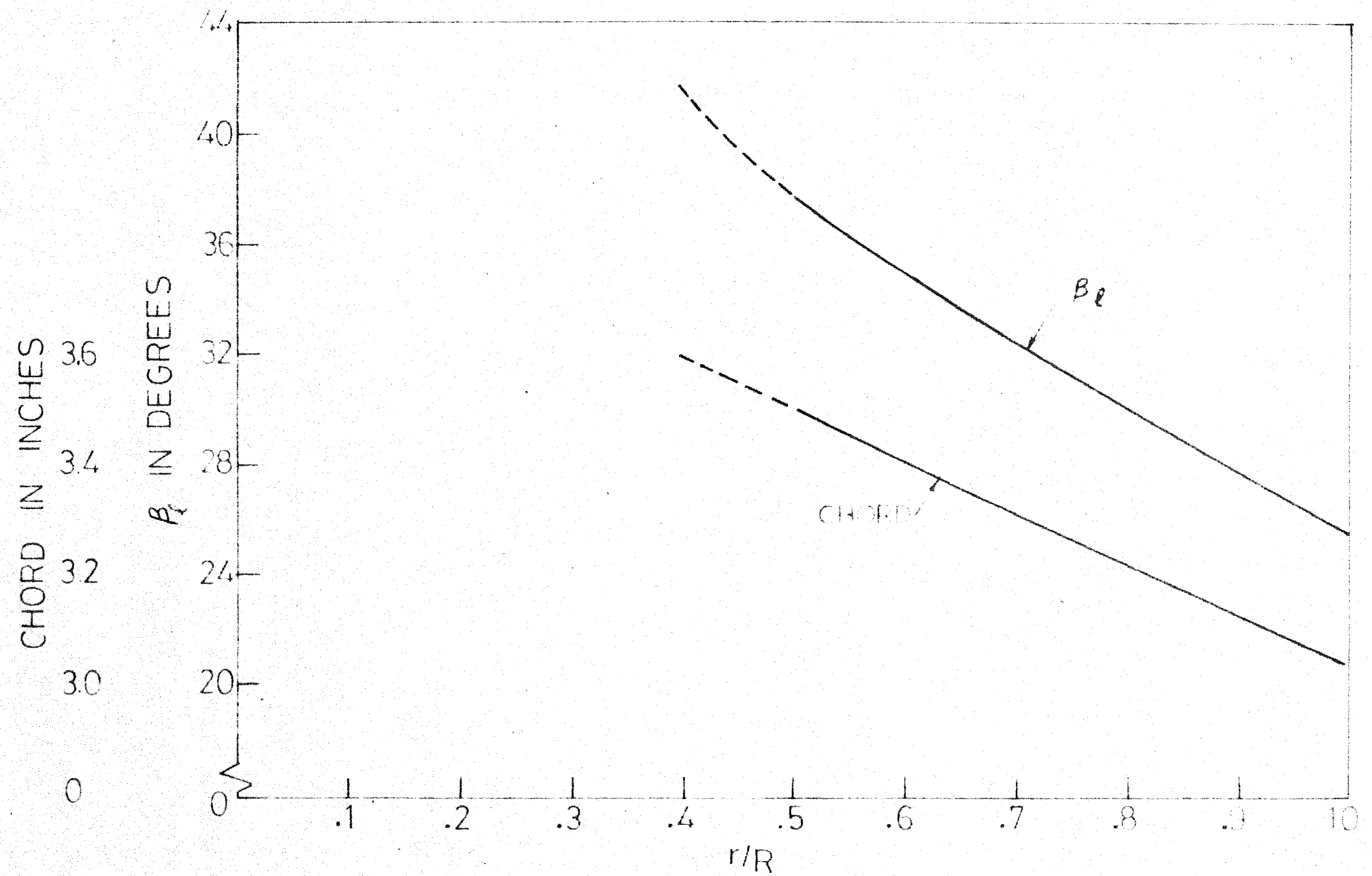
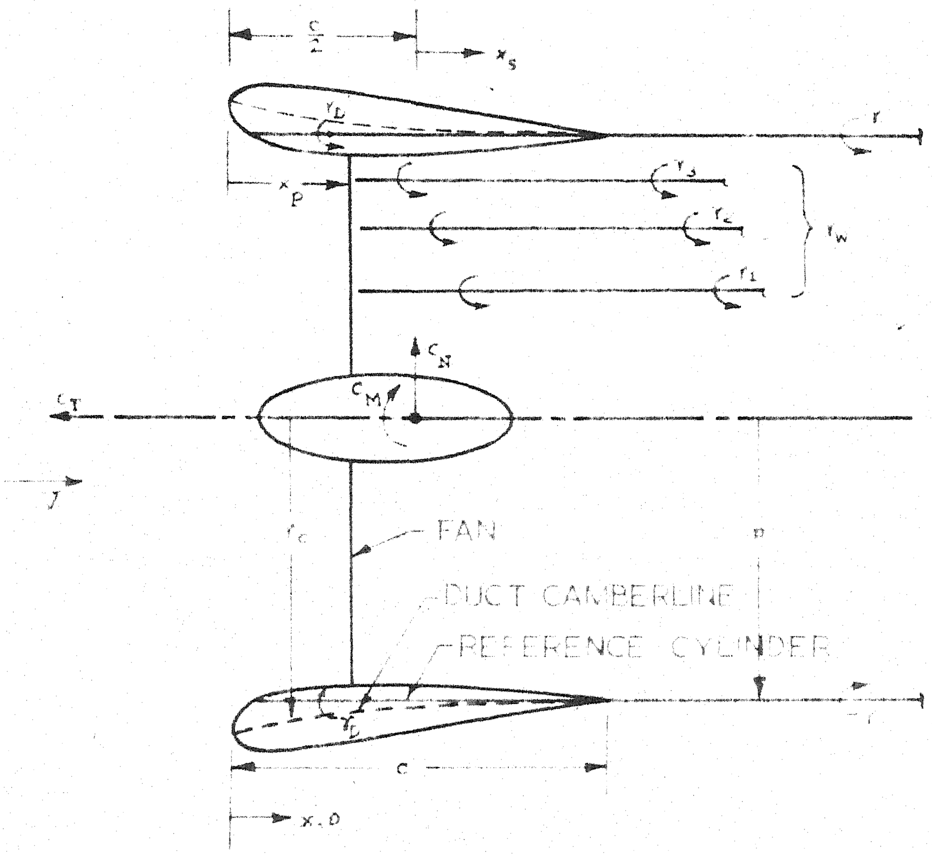
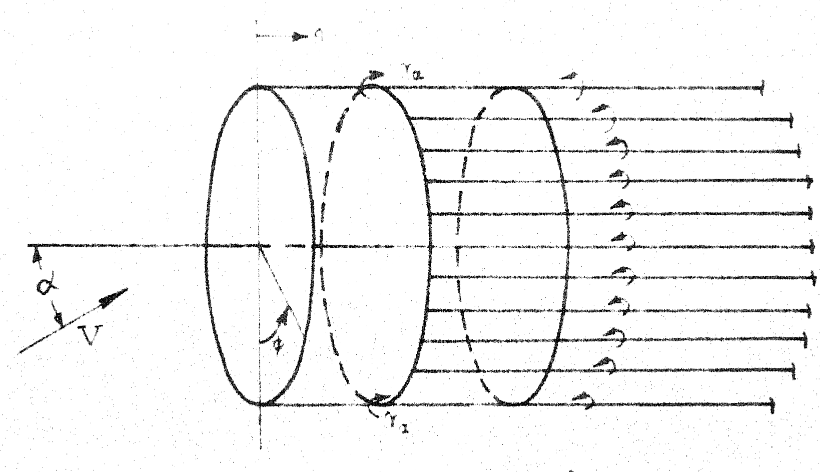


FIG. 7_ BLADE FORM CURVE OF THE PROPELLER



(a) DUCT FAN CENTERBODY IN AXIAL FLOW



(b) DUCT AT ANGLE OF ATTACK

FIG. 8 _ FLOW MODELS FOR DUCTED FAN

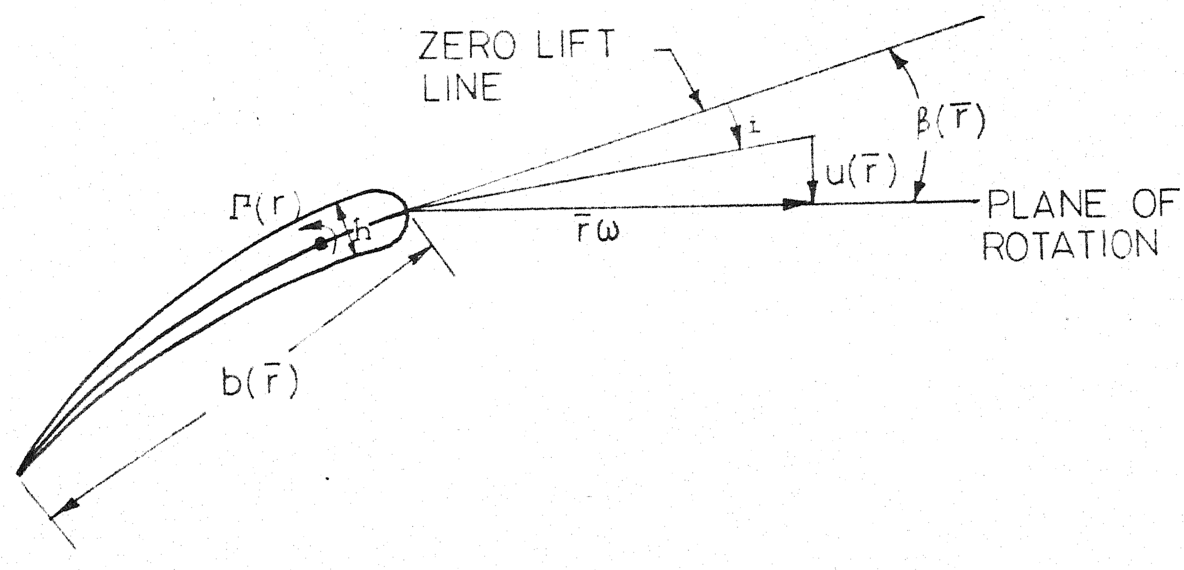


FIG. 9 _ BLADE SECTION FLOW CHARACTERISTICS.

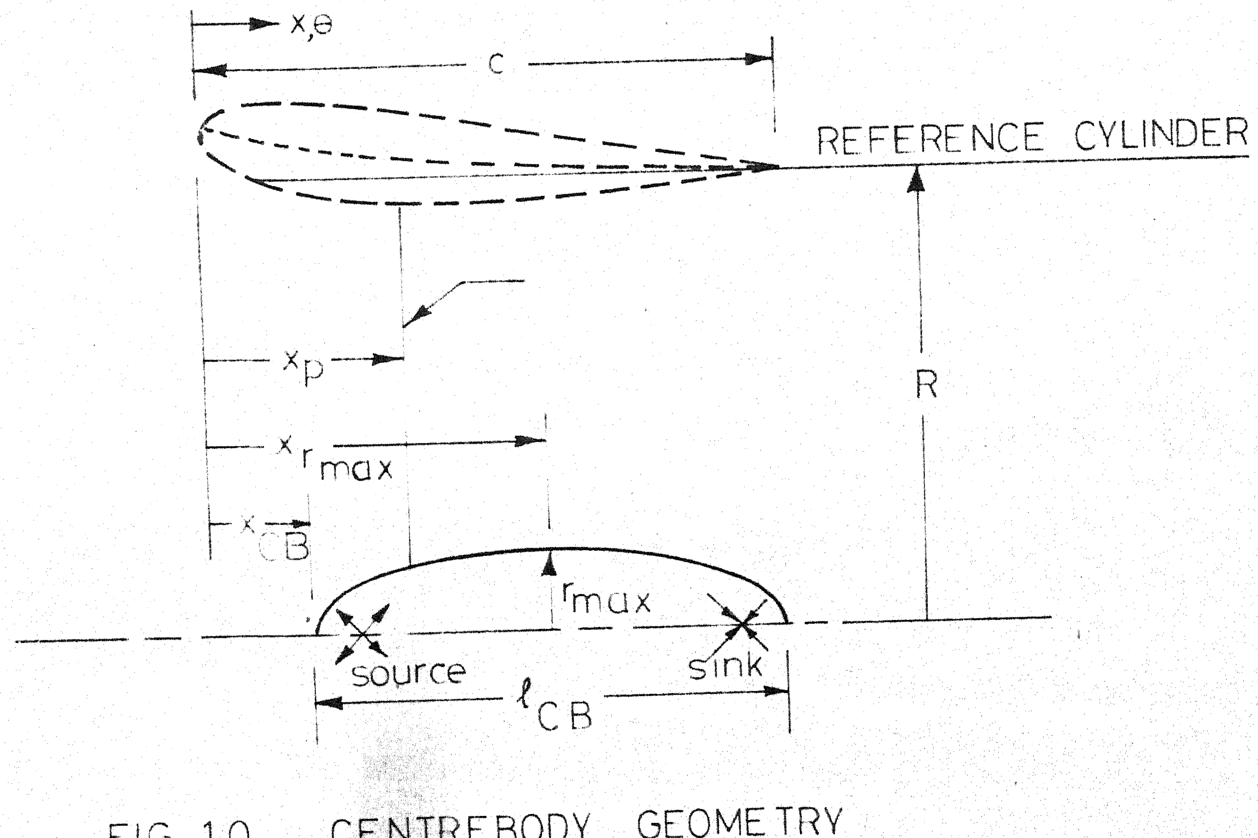


FIG. 10 _ CENTREBODY GEOMETRY

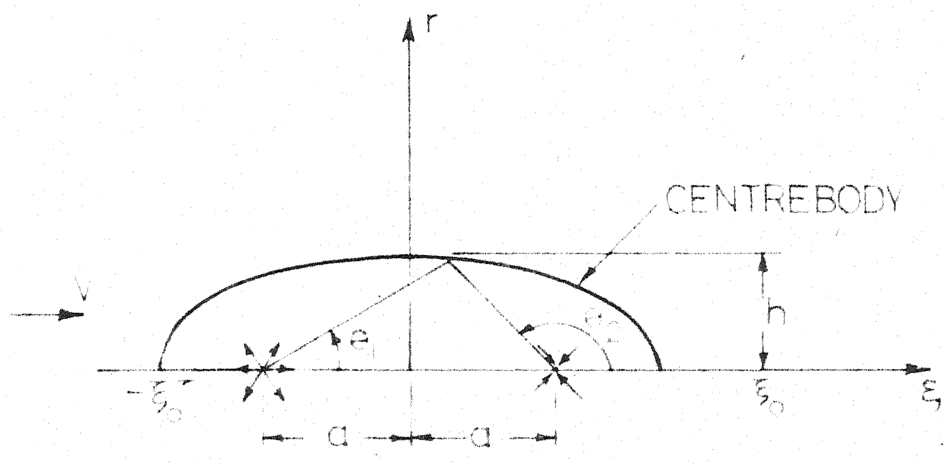


FIG. 11_ NOTATIONS USED IN DERIVING THE VELOCITY COMPONENTS INDUCED BY THE CENTREBODY

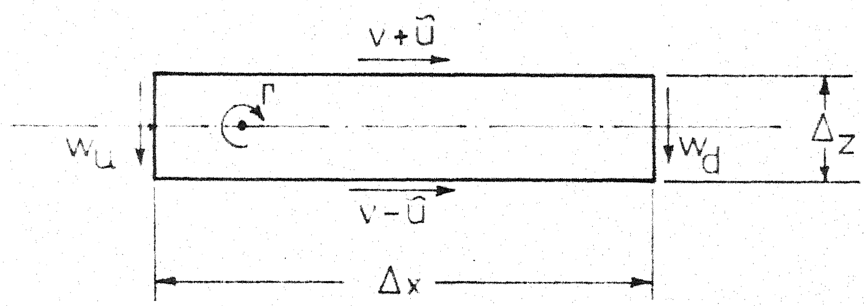


FIG.14_NOTATIONS USED IN GETTING THE RELATIONSHIP BETWEEN Γ & C_p

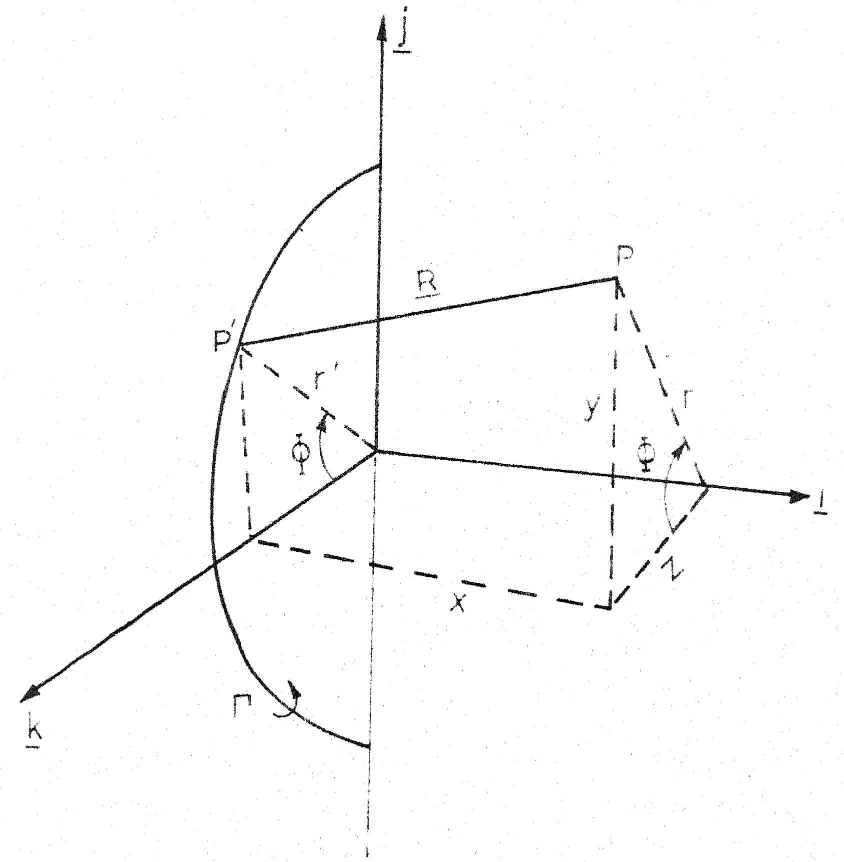


FIG. 12 _ NOTATION USED IN DERIVING THE VELOCITY COMPONENTS INDUCED BY A VORTEX RING

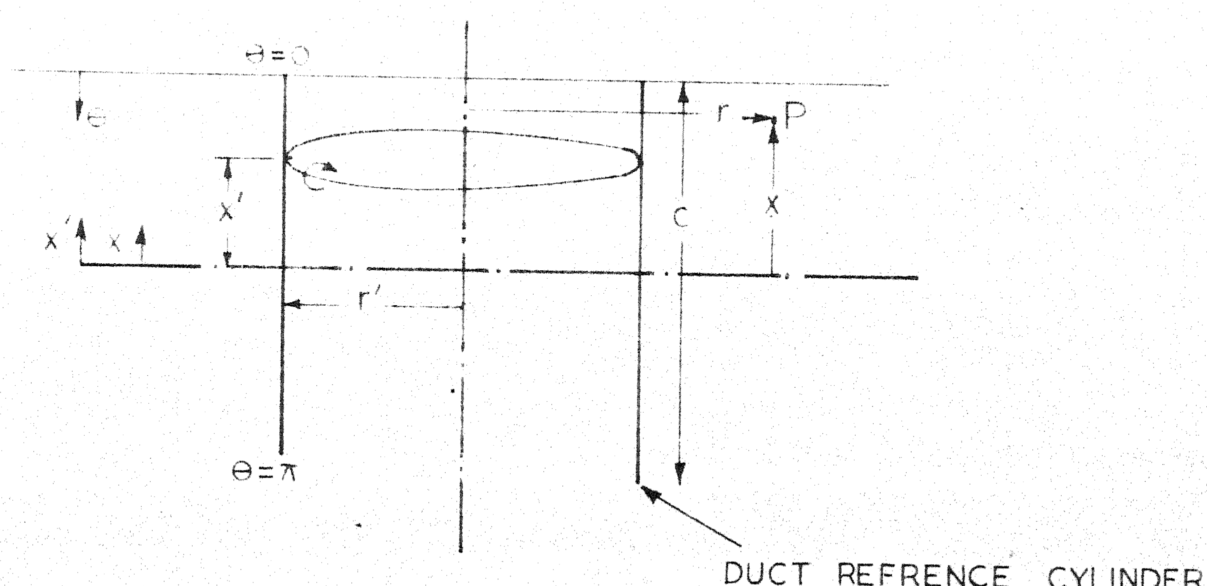
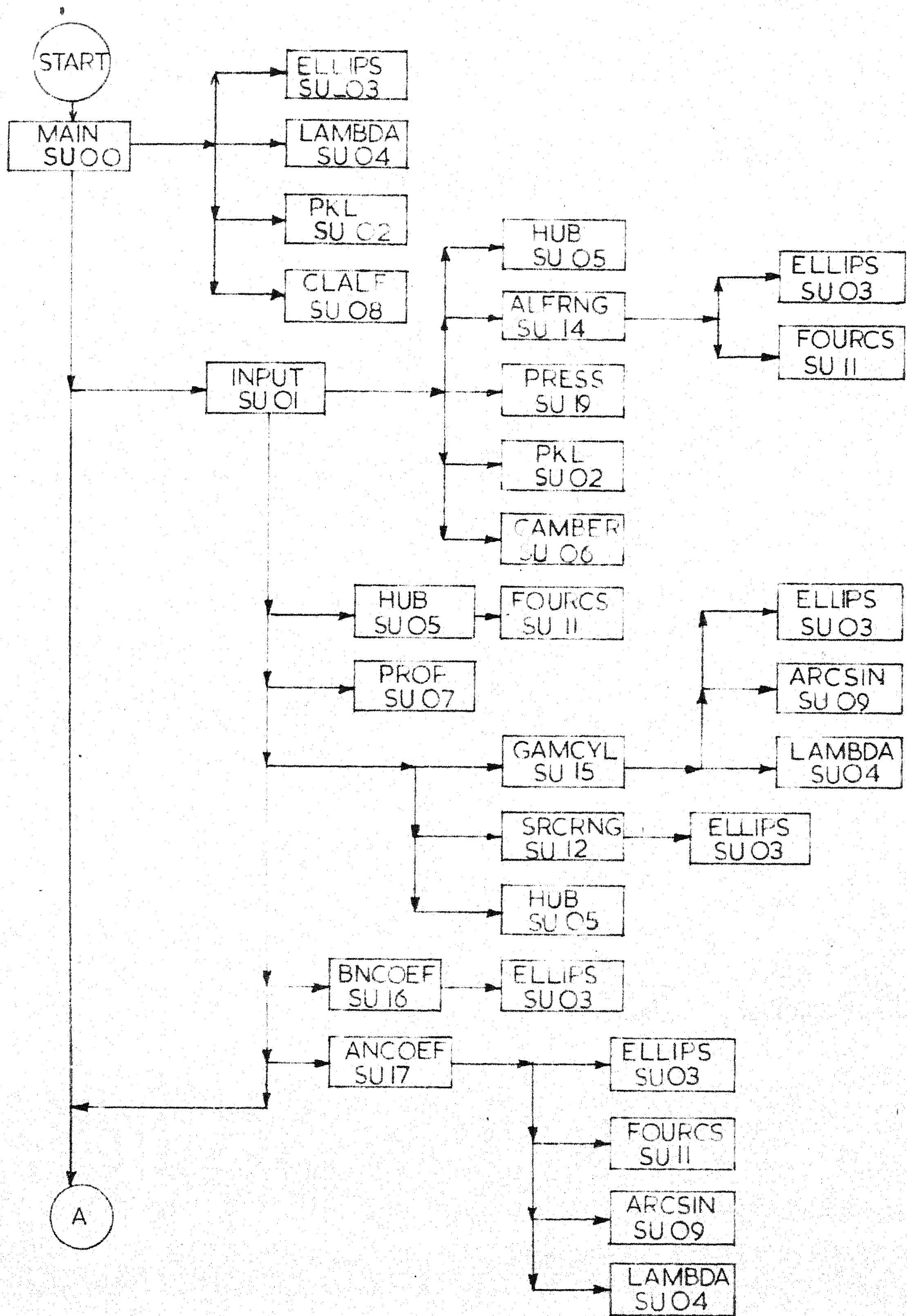
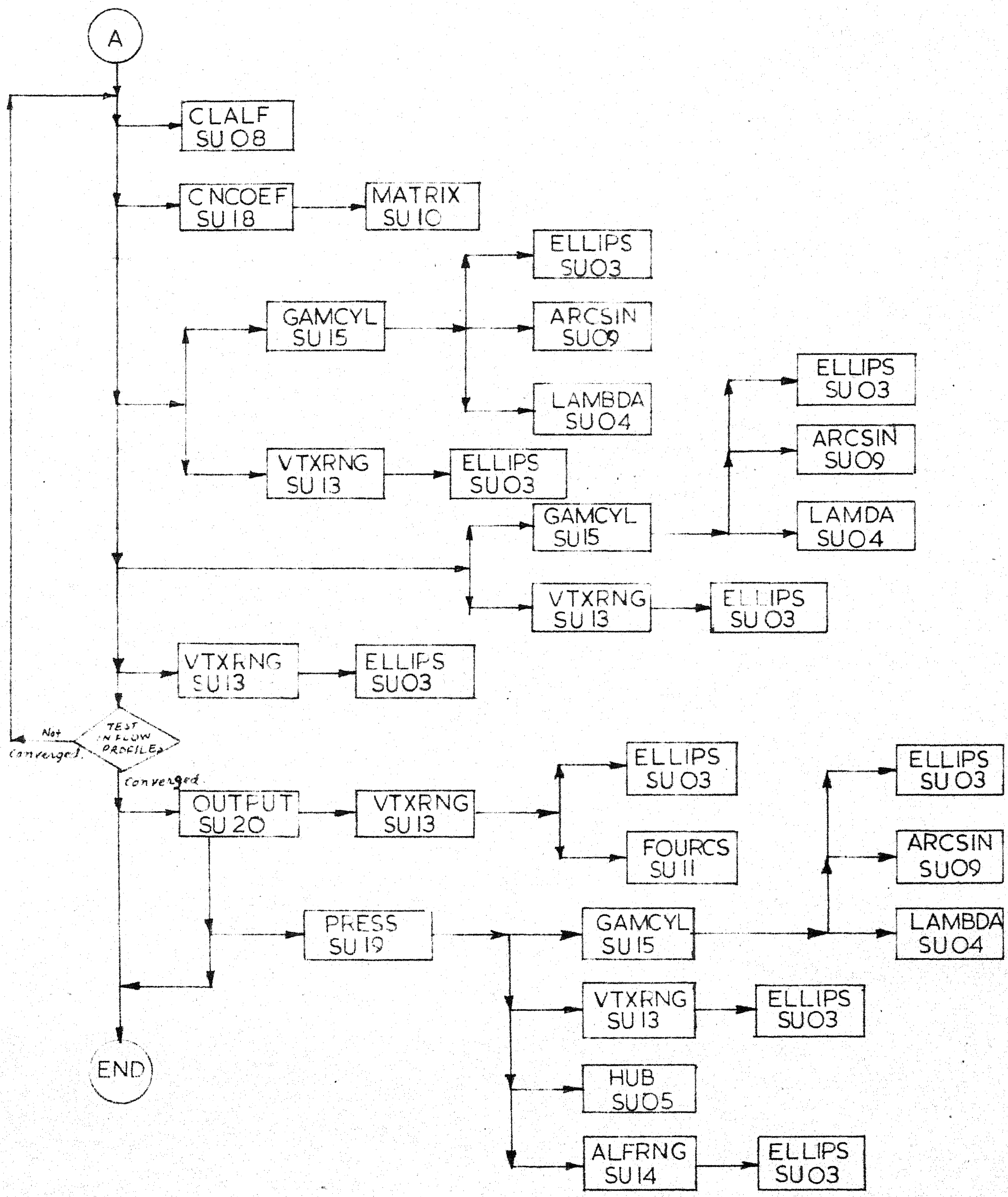


FIG.13 _ NOTATION USED IN DERIVING THE INDUCED VELOCITY COMPONENTS DUE TO THE DUCT BOUND VORTICITY



(a) INITIALIZATION SECTION

FIG.15_RELATIONSHIP BETWEEN SUBPROGRAMS OF PART I



(b) ITERATION AND CONCLUSION SECTION

FIG.15_CONCLUDED

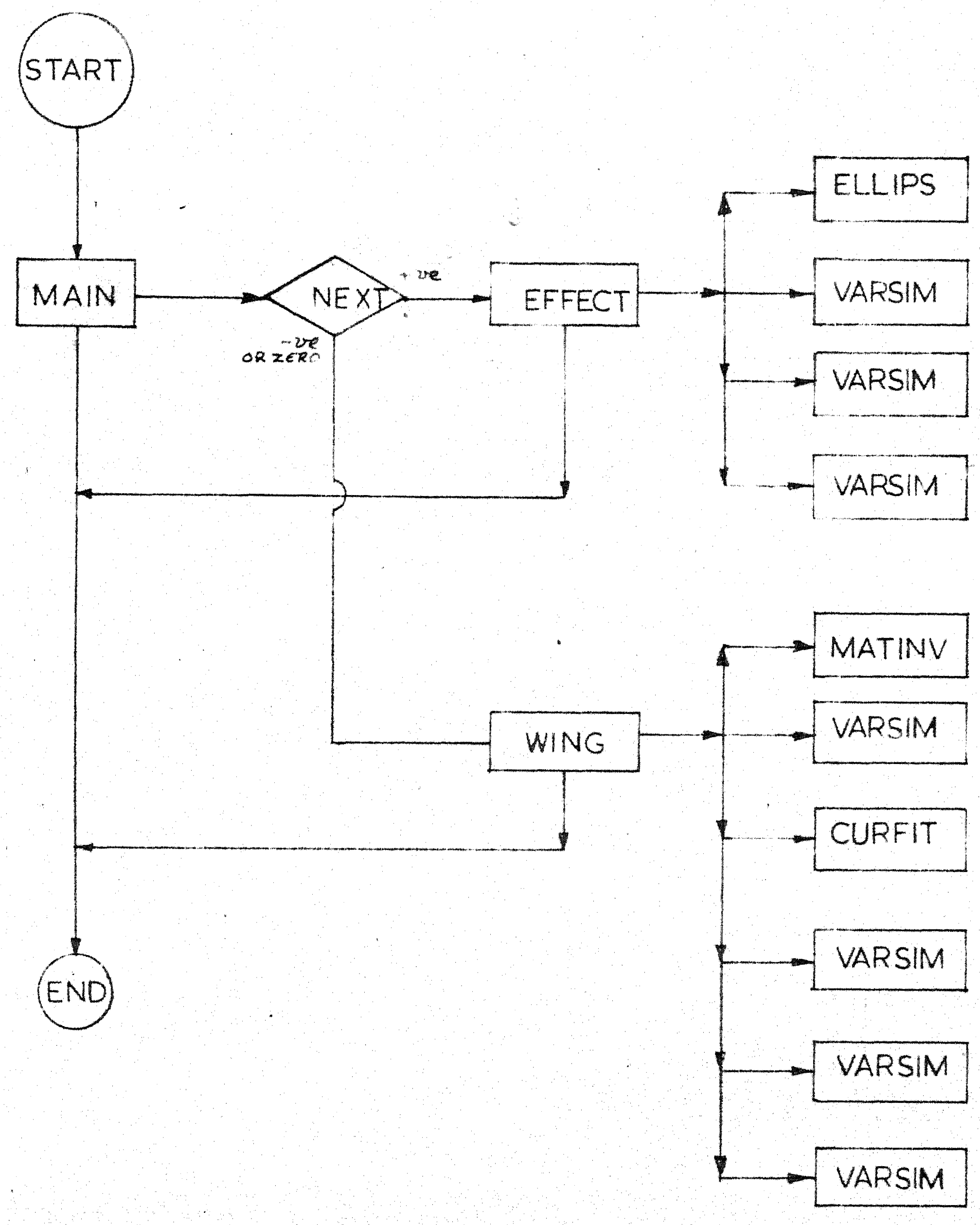


FIG. 16_RELATIONSHIP BETWEEN SUBPROGRAMS OF PART II

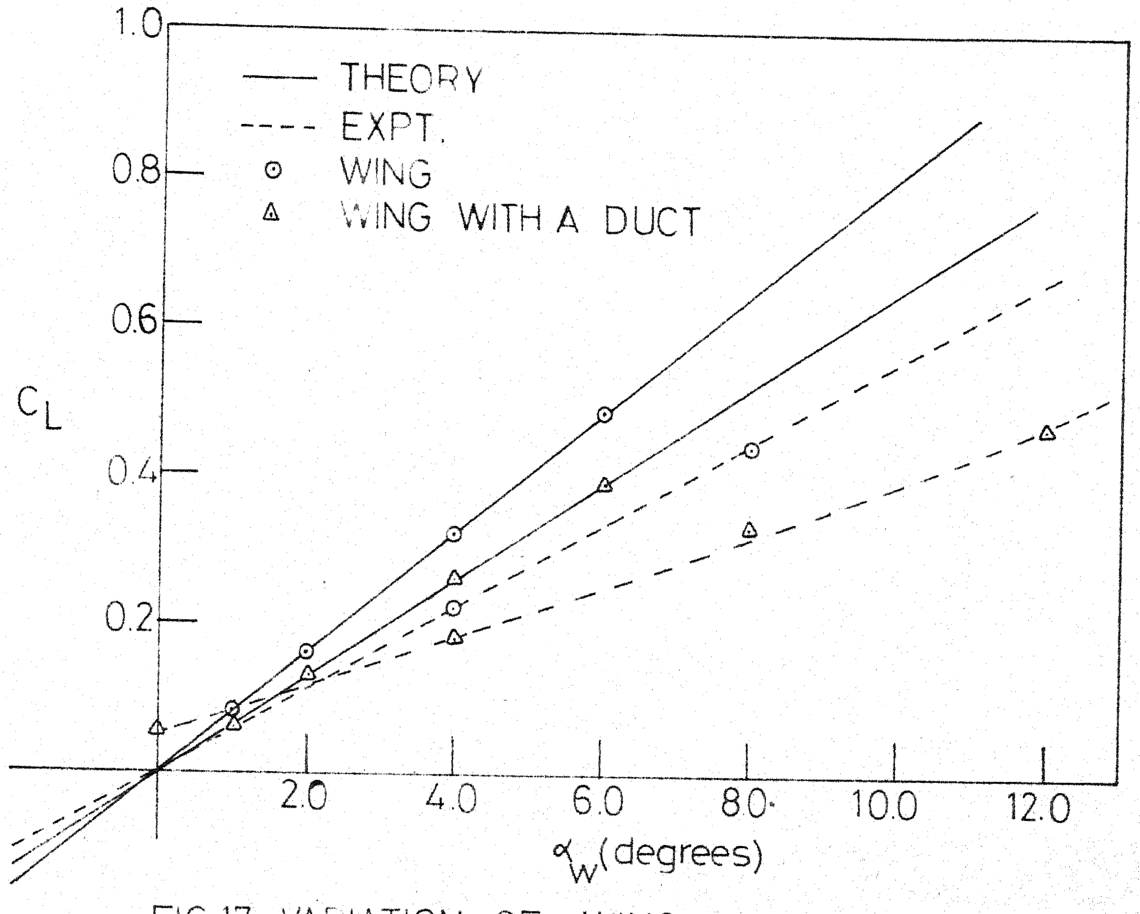


FIG.17_ VARIATION OF WING AND WING WITH A DUCT C_L WITH WING ANGLE OF ATTACK.

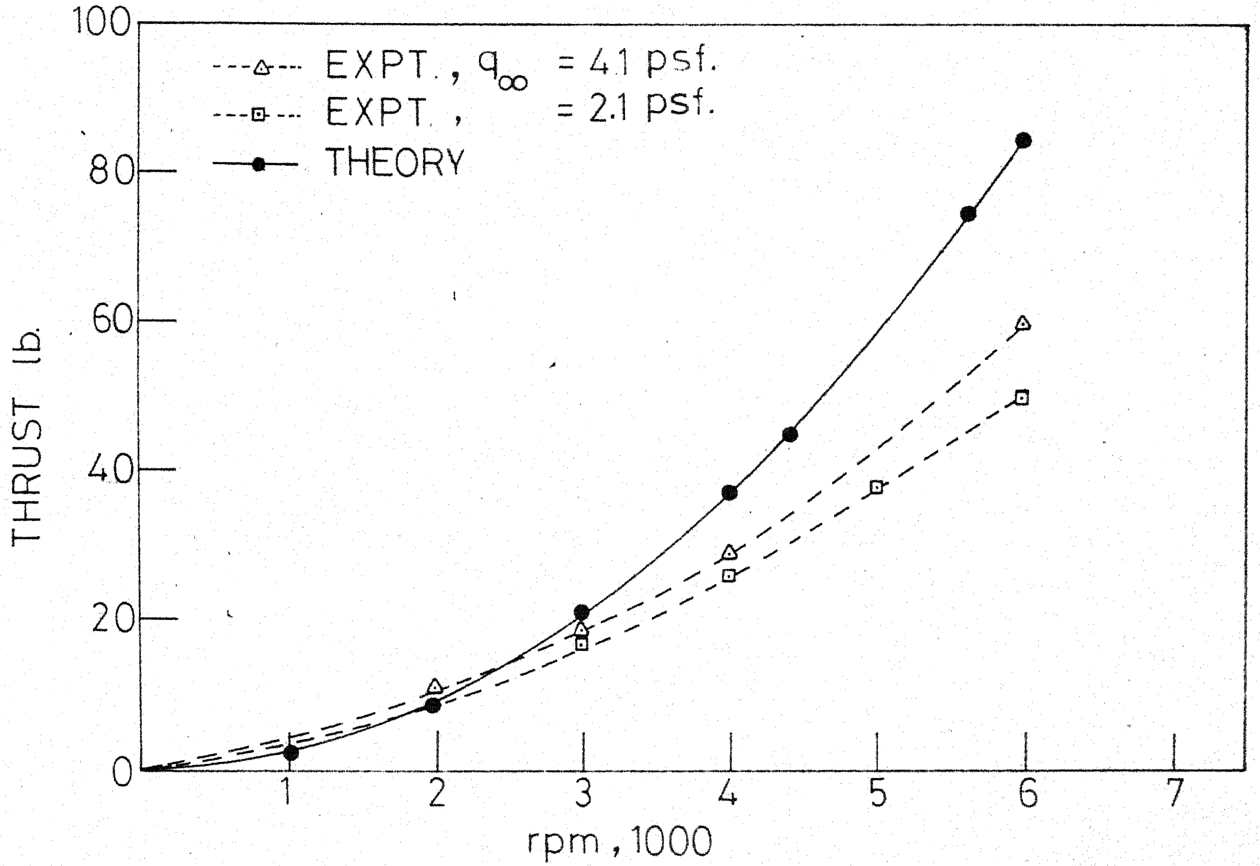


FIG.18 _ VARIATION OF FAN THRUST WITH FAN rpm.

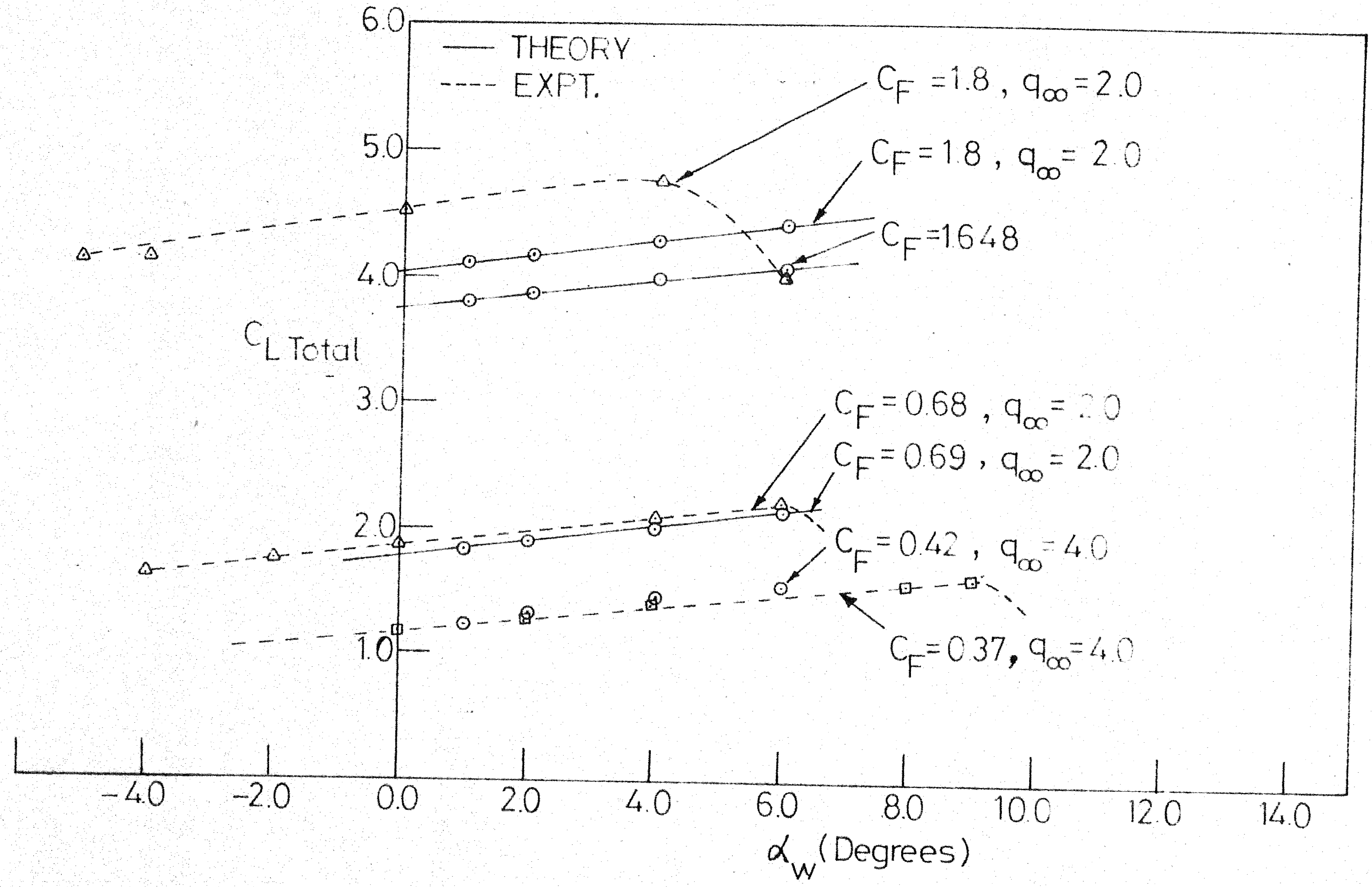


FIG.19 _ VARIATION OF FAN - IN - WING TOTAL LIFT COEFFICIENT WITH WING ANGLE OF ATTACK .

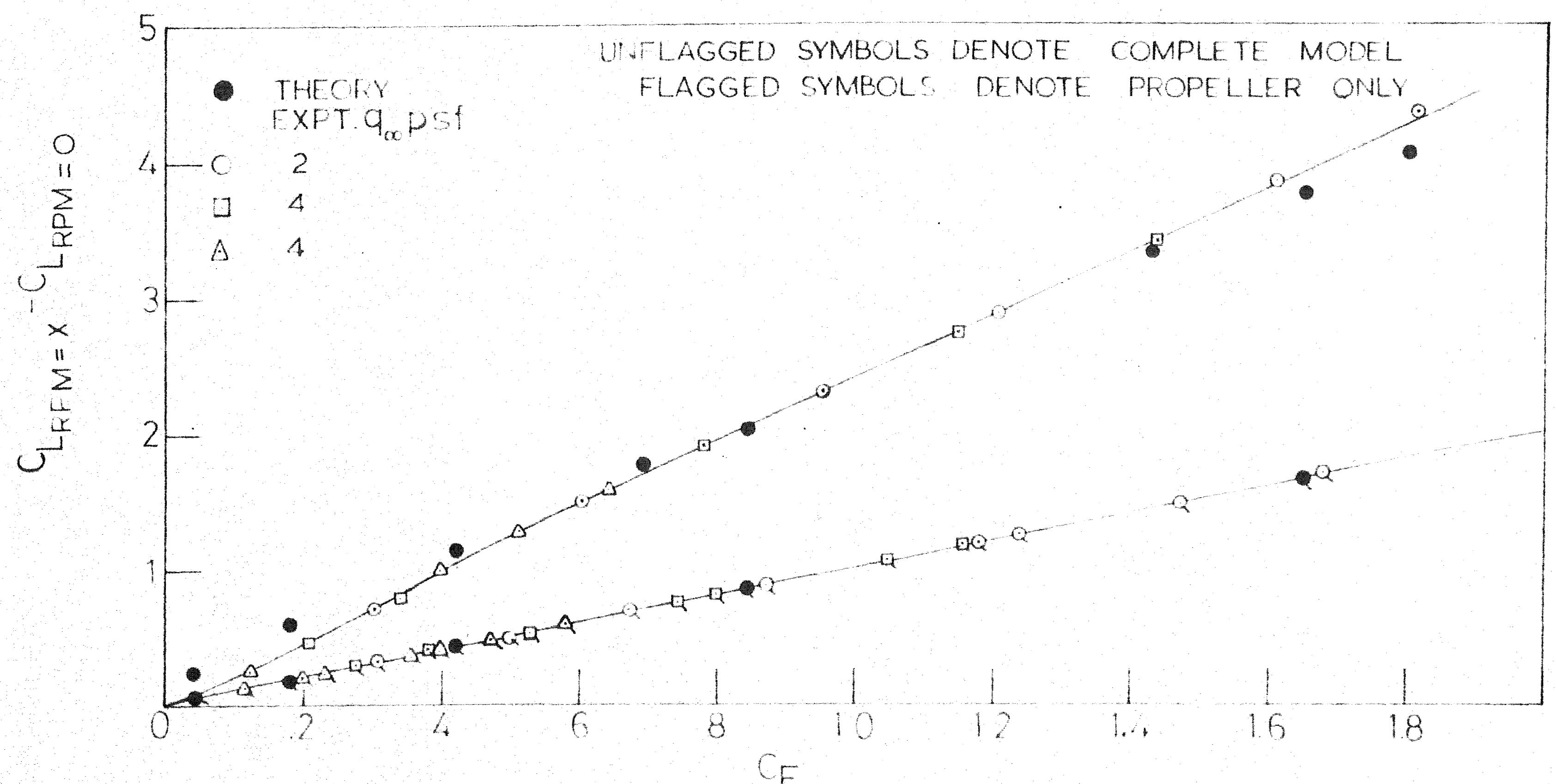


FIG.20_VARIATION OF INCREMENTAL LIFT COEFFICIENT WITH PROPELLER LOADING , $\alpha = 0$

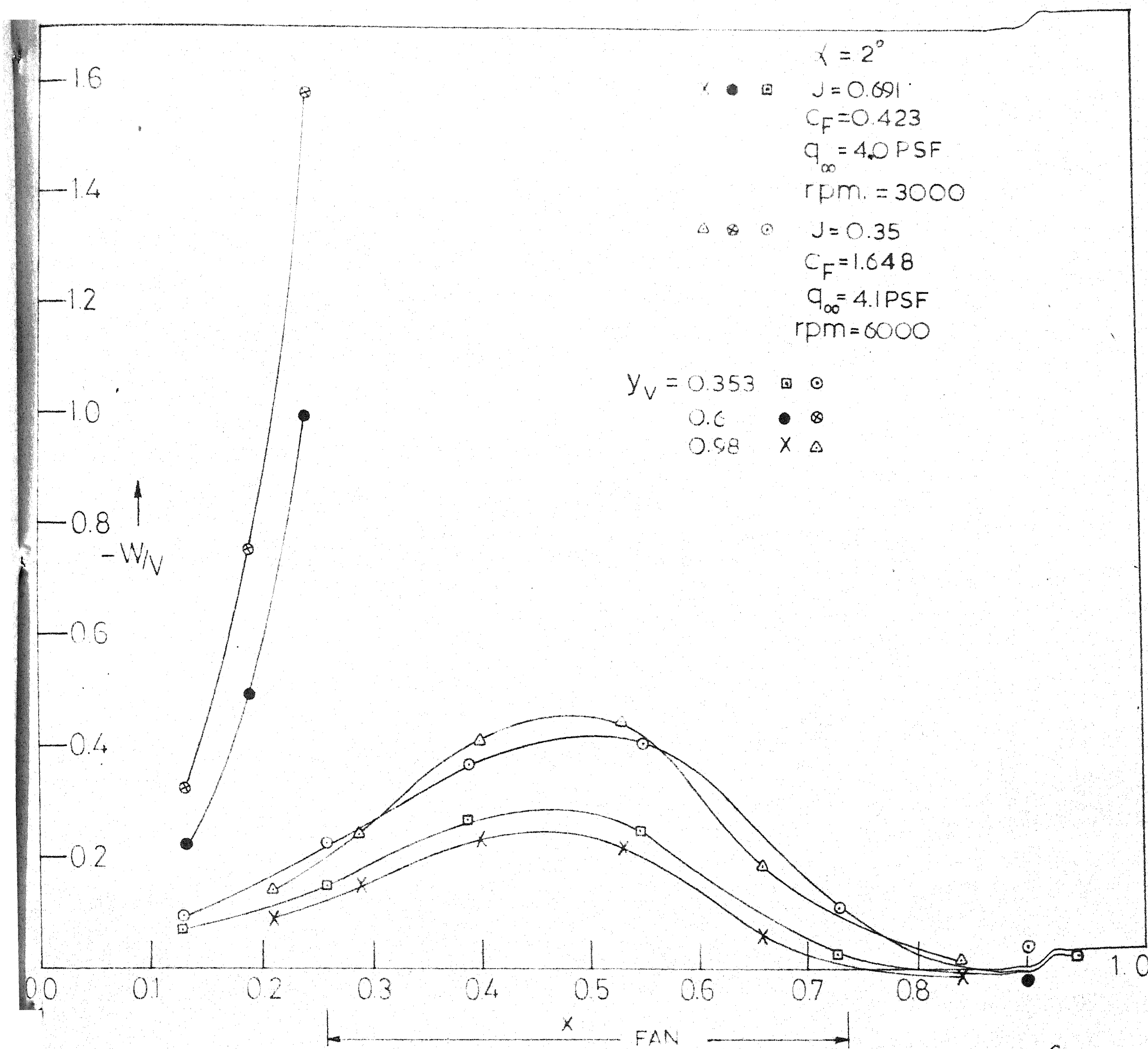


FIG. 21_ FAN INDUCED UPWASH DISTRIBUTION ON THE WING

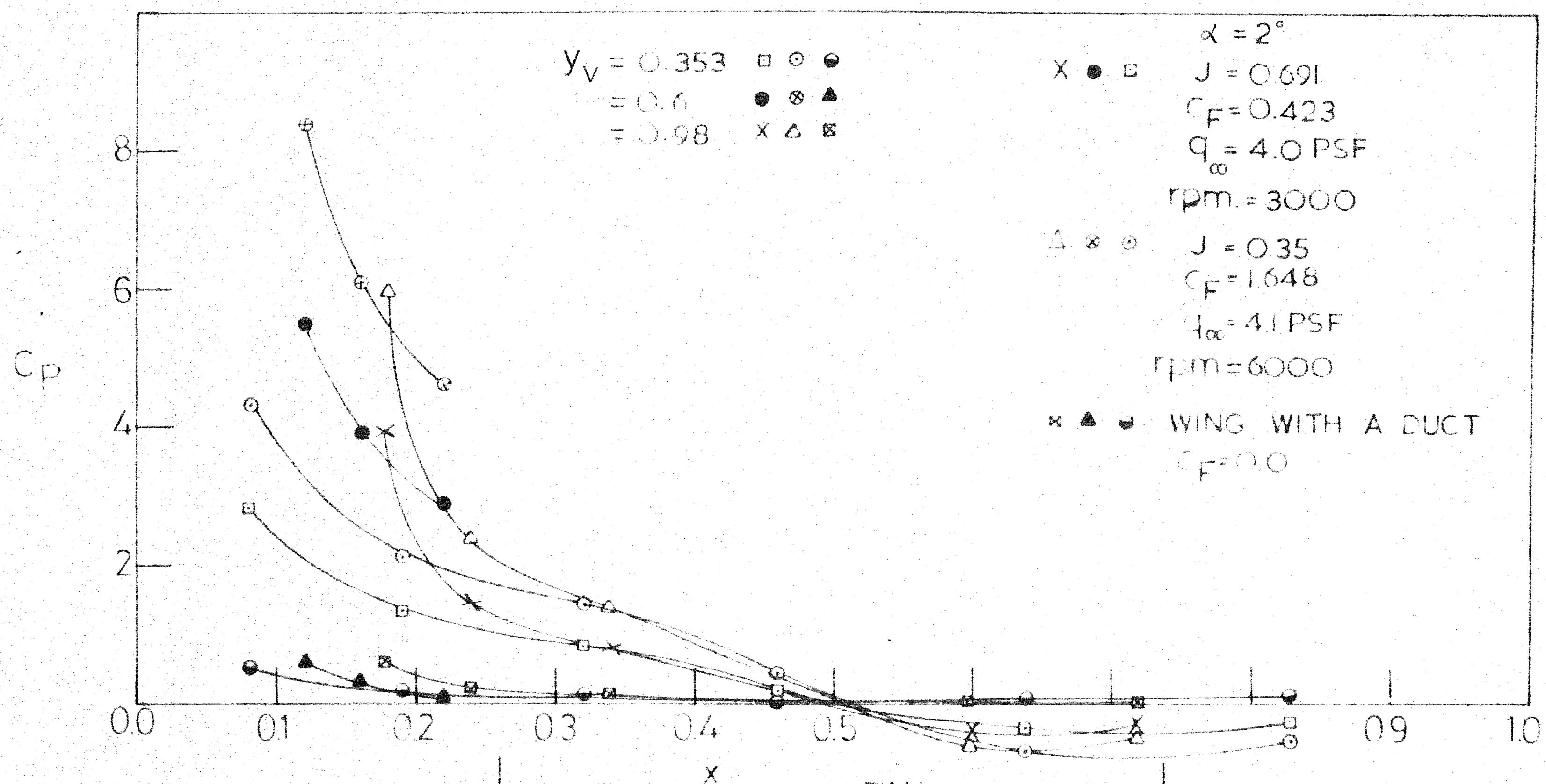


FIG.22_PRESSURE DISTRIBUTION FOR THE FAN-WING SYSTEM